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5. Completing partial latin boxes
Definition 1

A partial latin square of order n is an $n \times n$ array of n symbols so that each symbol appears at most once in each row and column.
**Definition 1**

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\[
\begin{array}{ccc}
1 & 3 & 5 \\
4 & 3 & \\
1 & 3 & \\
3 & 1 & \\
3 & 4 & 1
\end{array}
\]
Definition 1

A partial latin square of order $n$ is an $n \times n$ array of $n$ symbols so that each symbol appears at most once in each row and column.
Theorem 1
(Smetaniuk, 1981) Every partial latin square of order $n$ with at most $n - 1$ entries is completable.
Definition 2

A partial Latin square $P$ of order $n$ is called avoidable if there is a Latin square $L$ of order $n$ such that on every set of $n$ symbols $L$ contains no part of $P$. 
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$$
\begin{array}{ccc}
1 & 2 & 3 \\
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4 & 1 & 2 \\
3 & 4 & 2 \\
\end{array}
$$
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\[
\begin{array}{ccc}
1 & 2 & 3 \\
1 & 2 & 3 \\
4 & 1 & \\
5 & 2 & \\
3 & 4 & \\
\end{array}
\quad
\begin{array}{cccc}
2 & 3 & 5 & 1 & 4 \\
1 & 2 & 4 & 5 & 3 \\
5 & 1 & 3 & 4 & 2 \\
3 & 4 & 1 & 2 & 5 \\
4 & 5 & 2 & 3 & 1 \\
\end{array}
\]
Theorem 1

Every partial Latin square of order \( k \geq 4 \) is avoidable.
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Definition 3

Let $P$ and $Q$ be partial latin squares of order $n$ that avoid each other. We say that $P$ is strictly completable with respect to $Q$ if $P$ can be completed to a Latin square $L$ and $L$ avoids $Q$. 
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Conjecture 1

Let $P$ and $Q$ be partial latin squares of order $n > 3$ that avoid each other. If $P$ contains at most $n - 2$ entries, then $P$ can is strictly completable with respect to $Q$. 
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1 & 2 & 3 \\
\hline \\
\hline \\
\end{array}
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\[
\begin{array}{ccc}
1 & 2 & 3 \\
\hline
\end{array}
\quad
\begin{array}{c}
4 \\
\hline
\end{array}
\]
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Theorem 2

Let \( k = 4t \) for \( t \geq 9 \) be a positive integer.

Let \( P \) and \( Q \) be partial latin squares of order \( k \) such that \( P \) and \( Q \) avoid each other. If \( P \) contains at most \( t-1 \) entries, then \( P \) can be strictly completed with respect to \( Q \).
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Lemma 3

Let $P$ and $Q$ be partial latin squares of order 4 that avoid each other and let $P$ contain at most one entry. Then $P$ can be strictly completed with respect to $Q$ provided

1. $Q$ contains at most 3 symbols, or
2. $Q$ contains 4 symbols of which at least one appears only once.
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A 4-tuple of symbols is called bad in $X$ if each symbol in the 4-tuple appears at least twice in $X$. 
Main Result

preliminary results

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**Lemma 4**

Let $x$ be a symbol appearing in $X$. There are at most 20 bad 4-tuples in $X$ containing $x$. 
Main Result

Theorem of Daykin and Häggkvist

Theorem 5

Let $0 \leq d < k$ and let $H$ be an $r$-partite $r$-uniform hypergraph with minimum degree $\delta(H)$ and $|V(H)| = rk$. If

$$\delta(H) > \frac{r-1}{r} \left( k^{r-1} - (k - d)^{r-1} \right),$$

then $H$ has more than $d$ independent edges.
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Proof outline for main result

Let $T$ be a partial latin square of order $t$ on the symbol set $\{X_1, \ldots, X_t\}$ such that

$$
\begin{array}{c|c|c|c}
X_1 & X_2 & X_3 \\
X_4 & X_5 & X_6 \\
X_7 & X_8 & X_9 \\
\end{array}
$$

Since $T$ contains at most $t-1$ entries, $T$ can be completed.
Let $T$ be a partial latin square of order $t$ on the symbol set $\{X_1, \ldots, X_t\}$ such that

$\text{cell } (j, l) \text{ contains } X_i \text{ if and only if } i \text{ appears in the corresponding } 4 \times 4 \text{ subsquare of } P.$
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We wish to find a partition \( S_1, S_2, \ldots, S_t \) of \([4t]\) such that

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2. $B = \{t + 1, \ldots, 2t\}$,
3. $C = \{2t + 1, \ldots, 3t\}$, and
4. $D = \{3t + 1, \ldots, 4t\}$. 

The edge $((X_i, i), b, c, d)$ is included in $H$ if and only if $\{i, b, c, d\}$ is not a bad 4-tuple for each 4 $\times$ 4 subsquare of $Q$ corresponding to $X_i$ in $T$. 

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Proof outline for main result

\[ d_H((X_i, i)) \geq t^3 - 20t \]

According to the theorem of Daykin and Häggkvist, \( H \) has \( t \) independent edges provided \( \delta(H) > \frac{3}{4}(t^3 - 1) \).

Theorem 6

Let \( 0 \leq d < k \) and let \( H \) be an \( r \)-partite \( r \)-uniform hypergraph with minimum degree \( \delta(H) \) and \( |V(H)| = rk \). If \( \delta(H) > r - 1 \frac{kr}{r} - 1 - (k - d) \frac{r}{r} - 1 \), then \( H \) has more than \( d \) independent edges.
Proof outline for main result

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\[ \delta(H) > \frac{r - 1}{r} \left( k^{r-1} - (k - d)^{r-1} \right) \]

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\[
\delta(H) \geq t^3 - 20t > \frac{3}{4}(t^3 - 1)
\]
Proof outline for main result

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Let \( \{e_1, \ldots, e_t\} \) be \( t \) independent edges in \( H \) where 
\[ e_i = ((X_i, i), b, c, d). \]
Proof outline for main result

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Let \( \{e_1, \ldots, e_t\} \) be \( t \) independent edges in \( H \) where \( e_i = ((X_i, i), b, c, d) \).

Set \( S_i = \{i, b, c, d\} \) for each \( i \).
Completing partial latin boxes

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Theorem 7

Let $t \geq 9$. Let $P$ be a $2 \times 4t \times 4t$ partial latin box with at most $2t - 1$ entries. Then $P$ can be completed.
THANK YOU FOR YOUR ATTENTION!