Math 501/Quiz 2

This quiz is due at the beginning of class on Tuesday, September 13. Since this is a take home quiz, there will be little to no partial credit. Show all your work clearly. There are more problems on the back.

1. Decide if the following statements are True or False. 10 points for each part.

   • If \( A \subseteq \mathbb{R} \) is not dense in \( \mathbb{R} \) and \( B \subseteq \mathbb{R} \) is not dense in \( \mathbb{R} \), then \( A \cup B \) is not dense in \( \mathbb{R} \).

   • If \( A \cup B \) is uncountable, then both \( A \) and \( B \) are uncountable.
2. (20 pts) Show that the set of all intervals with rational endpoints is countable.

3. (Bonus, 10 pts) Let $A \subseteq \mathbb{R}$ be uncountable. Show that there is $p \in A$ such that every open interval containing $p$ contains uncountably many points of $A$. 