1. Determine whether the following statements are true or false. Give a complete justification for your answer. You will receive 4 pts for each correct answer and 4 pts for a correct explanation.

- Let $G$ be a group. Then, for all $a, b \in G$, $(ab)^2 = a^2b^2$.

- $\mathbb{U}_{30}$ has 8 elements.

- $6x = 1 mod(63)$ has a solution over $\mathbb{Z}$.

- There are integers $u, v$ such that $7u + 6v = 2$. 
• $x^2 + 1 = 0$ has a solution over $\mathbb{Z}_{11}$.

2. Use the Euclid’s algorithm to find $(3234, 39)$. Then, write the gcd as $3234u + 39v$. Show all your steps.

3. Find the least nonnegative residue of $4^{925} \pmod{59}$. (You may have to apply modular arithmetic repeatedly.)
4. Let $a_1, a_2, \ldots, a_n \in \mathbb{Z}$. Show that 
\[(a_1 \cdot 10 + a_2 \cdot 10^2 + \ldots a_n \cdot 10^n) = (a_1 + a_2 + a_3 + \ldots a_n) \pmod{9} \).

5. Suppose $G$ is a group and $a, b, c, x \in G$. Solve for $x$ in the following equation. 
\[a^2ba^3xcb^4 = c.\]

6. Recall $\mathbb{Q}$ is the set of all rationals. For each $x, y \in \mathbb{R}$, define $x \sim y$ iff $x - y \in \mathbb{Q}$. Show that $\sim$ is an equivalence relation over $\mathbb{R}$. What is $[\sqrt{2}]$?
7. Consider \( \mathbb{Z} \), the integers, with operation \( \ast \) defined as follows: \( a \ast b = ab + a + b \). Is \((\mathbb{Z}, \ast)\) a group? Give a complete justification for your answer.

8. Extra Credit Problem for the Undergrads and required problem for the Grads.
Suppose \( a, b, c \in \mathbb{Z} \) such that \( a^2 + b^2 = c^2 \). Show that at least one of \( a, b, c \) is divisible by 3.