SHOW ALL WORK!!! NO CREDIT OTHERWISE!!!

1. Determine whether the following statements are true or false. Give complete justification for your answer.
   - Let $G$ be a finite group. Then, there is a positive integer $n$ such that $a^n = e$ for all $a \in G$.
   - If $\sigma$ is a permutation of order 3, then $\sigma$ is an even permutation.
   - $(1,2,3,4)$ is not the product of 3-cyles.
   - If $\alpha, \beta \in S_n$, then $\alpha \beta \alpha^{-1}$ is an even permutation.
   - $S_7$ has $7!$ elements of order 7.
   - Every two abelian groups of the same order are isomorphic.
   - Upto isomorphism, there is only one cyclic group of order $n$.
   - $\mathbb{Z}_2 \times \mathbb{Z}_8 \cong \mathbb{Z}_{16}$.
   - $(\mathbb{R}^+, \cdot) \cong (\mathbb{R}, +)$.
   - Every automorphism on a group $G$ is an inner automorphism on $G$.
   - $\text{Aut}(\mathbb{Z}_8)$ is cyclic.
   - Let $G$ be an abelian group of order 22. Then, $G$ is cyclic.
   - If $G$ is a nonabelian group of order 62, then it is isomorphic to some dyhedral group.
   - $\text{Aut}(\mathbb{Z}) \cong \mathbb{Z}_2$.
   - Let $G$ be a group and $H, K$ be two subgroups of $G$ of order 12 and 35. Then, $H \cap K$ has only one element.
   - Suppose that $G$ and $H$ are groups and $G \oplus H$ is cyclic. Then, $o(G)$ and $o(H)$ are relatively prime.
   - If you understand all subgroups of every permutation group, then you understand the group theoretic secret of the universe. (Cayley’s Theorem.)

2. BE SURE TO GO OVER ALL THE ASSIGNED HOMEWORK PROBLMEMS AND QUIZZES. I AM LISTING SOME MORE PROBLEMS BELOW.

3. Page 111-. Problems 9, 18, 28, 39.

4. Page 130-. Problems 5, 15, 33, 41.

5. Page 145-. 1,5,7, 19, 25,26, 36.

6. Page 162-. All the assinged problems.