1. Determine whether the following statements are true or false. Give a complete justification for your answer. You will receive 4 pts for each correct answer and 3 pts for a correct explanation.

- $S_5$ is cyclic.

- $\mathbb{Z}_{80}$ has four elements of order 8.

- Every infinite cyclic group has infinitely many generators.

- There is a finite group $G$ and $a, b \in G$ such that $o(ab) \neq o(a) \cdot o(b)$. 
• Let $G$ be a group and $Z(G)$ be the center of $G$. $Z(G) = G$ iff $G$ is abelian.

2. Prove that every cyclic group is abelian.

3. Let $G$ be a group and $a, x \in G$. Show that $o(xax^{-1}) = o(a)$. 

4. Consider the following permutation.

\[
\sigma = \begin{bmatrix}
1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\
2 & 3 & 4 & 5 & 6 & 1 & 8 & 7
\end{bmatrix}
\]

- Write \( \sigma \) as the product of disjoint cycles and then as the product of 2-cycles.

- What is the order of \( \sigma \).

5. Decide if \([2]\) is a generator of \( \mathbb{U}(25) \).

6. Let \( G \) be a group and \( H \) be a subgroup of \( G \). Let \( N = \{x \in G : xhx^{-1} \in H \text{ for all } h \in H\} \). Show that \( N \) is a subgroup of \( G \).
7. 10 pts. Extra Credit Problem for the Undergrads and required problem for the Grads.
Let \( G \) be a group and \( a, b \in G \) such that \( o(a) = 10 \) and \( o(b) = 13 \).
Show that \(<a> \cap <b> = \{e\} \).

8. 10 pts. Bonus for everyone. Show that no group is the union of two proper subgroups.