SHOW ALL WORK!!! NO CREDIT OTHERWISE!

• Let $G$ be an abelian group and $H, K$ be two subgroups of $G$. Let

$$HK = \{hk : h \in H, k \in K.\}$$

Show that $HK$ is a subgroup of $G$.

• Find a homomorphism from $\mathbb{Z}_4$ onto $\mathbb{Z}_2$.

• Use the Cauchy’s Theorem and internal direct products to argue that every abelian group of order 35 is cyclic.