Show all work!!! No credit otherwise!!! More problems on the back!!

1. Determine whether the following statements are true or false. You will get 4 points for determining correctly whether the statement is true or false and 4 points for a correct explanation.

(a) If $\phi : \mathbb{Z}_{11} \rightarrow S$ is a ring homomorphism which is not 1-1, then $\phi(x) = 0$ for all $x \in \mathbb{Z}_{11}$.

(b) If $D_1$ and $D_2$ are integral domains, then $D_1 \oplus D_2$ is an integral domain.

(c) If the characteristic of a ring $R$ is 0, then $R$ is infinite.

(d) Let $R$ be a finite commutative ring with an identity. If $A$ is a prime ideal of $R$ then $A$ is a maximal ideal of $R$. 
(e) The field of quotients of $\mathbb{Z}_5[x]$ has exactly 5 elements.

2. Let $R = \mathbb{Z}_3[x]$ and $A = \langle x^2 + 1 \rangle$. Find the inverse of $(x + 2) + A$ in the quotient ring $R/A$.

3. Let $R$ be a ring and $A, B$ be ideals of $R$. Show that $AB = \{a_1b_1 + a_2b_2 + \ldots + a_nb_n : a_1, \ldots a_n \in A, b_1, \ldots b_n \in B, n \in \mathbb{N}\}$ is an ideal of $R$. 
4. Explain why there is no nontrivial ring homomorphism from \(\mathbb{Z}\) into \(2\mathbb{Z}\).

5. Is there a field with 12 elements? Give a complete justification for your answer.

Let \(\mathbb{Q}(\sqrt{5}) = \{ a + b\sqrt{5} : a, b \in \mathbb{Q} \} \) and \(\mathbb{Q}(\sqrt{2}) = \{ a + b\sqrt{2} : a, b \in \mathbb{Q} \}\). Show that \(\mathbb{Q}(\sqrt{5})\) and \(\mathbb{Q}(\sqrt{2})\) are group isomorphic but not ring isomorphic.