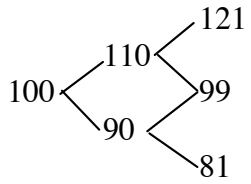
Ewa Kubicka http://www.math.louisville.edu/~ewa/
Course M seminar: http://www.math.ilstu.edu/actuary/prepcourses.html
Course M Casualty/Property Manual: http://www.neas-seminars.com/registration/

Practice Problem for exam MFE for the week after 01/26/08.

Consider the following information about a European call option on stock ABC:

- The strike price is $95.
- The current stock price is $100.
- The time to expiration is 2 years.
- The continuously compounded risk-free rate is 5% annually.
- The stock pays no dividends.
- The price is calculated using a 2-step Binomial Model where each step is one year in length.

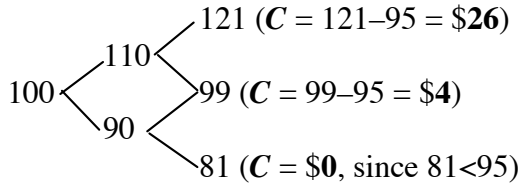
The stock price tree is shown below:



Calculate the price of the call on stock ABC.

(A) Less than 13.50
(B) At least 13.50 but less than 14.00
(C) At least 14.00 but less than 14.50
(D) At least 14.50 but less than 15.00
(E) At least 15.00

Solution. (Problem # 14 from CAS Exam 3, Sp.08)
The different values of the call at time 2 would be as follows:



We will compute the expected present value of these prices using risk neutral probabilities p^* and 1- p^*.
First, we compute constants u and d:

$u = \frac{110}{100} = 1.1$ (notice that $\frac{121}{110} = \frac{99}{90} = 1.1$ as well)

$d = \frac{90}{100} = 0.9$ (notice that $\frac{99}{110} = \frac{81}{90} = 1.1$ as well)

Now, we compute the risk neutral probability of an up move p^*:

$$p^* = \frac{e^{(r-\delta)h} - d}{u - d} = \frac{e^{0.5} - 0.9}{1.1} = 0.7564,$$

and the risk neutral probability of a down move:1- p^*= 1-0.7564 = 0.2436.

The expected present value at time 0 using risk neutral probabilities is:

$$Call\ price = e^{-2\cdot 0.05}\left[26\cdot p^*\cdot p^* + 4\cdot\left(1-p^*\right)\cdot p^* + 4\cdot p^*\cdot\left(1-p^*\right) + 0\cdot p^*\cdot p^*\right] =$$

$$= e^{-2\cdot 0.05}\left[26\cdot(0.7564)^2 + 4\cdot 2\cdot(0.2436)(0.7564)\right] = \mathbf{\$14.79}.$$

ANSWER:**D**

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