Ewa Kubicka http://www.math.louisville.edu/~ewa/
Course M seminar: http://www.math.ilstu.edu/actuary/prepcourses.html
Course M Casualty/Property Manual: http://www.neas-seminars.com/registration/

Practice Problem for exam MFE for the week after 09/08/07.
Assume the Black-Scholes framework. You are given:

- The strike price is $40.
- The put option expires in 2 year.
- $\delta = 0$.
- The stock is currently selling for $40
- $\sigma = 0.30$.
- The continuously compounded risk-free interest rate is 0.08.

The price of the compound call option giving you the right to pay $2 1 year from today to buy the above put option is $ 2.30.
What is the price of the compound option giving you the right to *sell* the above put option for $2 1 year from today?

(A) less than $0.30
(B) at least $0.30 but less than $0.35
(C) at least $0.35 but less than $0.40
(D) at least $0.30 but less than $0.45
(E) more than $0.45

Solution: **E.**

Our data is: S=\$40, K=\$40, σ=0.3, r=0.08, T=2, and δ=0.
CallOnPut(S=\$40, K=\$40, x, σ=0.3, r=0.08,t =1. T=2, δ=0) = 2.30, where x is the price of the standard put.

First we compute the price of the standard put.

$$P(S,K,\sigma,r,T,\delta) = Ke^{-rT}N(-d_2) - Se^{-\delta T}N(-d_1), \text{ where}$$
$$d_1 = \frac{\ln(S/K) + \left(r - \delta + \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}}$$
$$d_2 = d_1 - \sigma\sqrt{T}$$

$$d_1 = \frac{\left(0.08 + \frac{1}{2}0.3^2\right)2}{0.3\sqrt{2}} = 0.59.$$
$$d_2 = 0.59 - 0.3\sqrt{2} = 0.17$$

$x = P = 40e^{-0.08\times 2}N(-0.17) - 40N(-0.59) =$ (34.0858)(1- 0.5675) – (40)(1- 0.7224) = 3.64.

If we buy the above standard put and the compound option – put-on-put for A, then in a year we either keep the standard put or sell it for \$2. This is identical to putting $2e^{-0.08}$ in the risk free bond and buying the compound option – call-on-put, which price we know to be \$2.30.

Therefore, the following equation must be true:
$$3.64 + A = 2.30 + 2e^{-0.08}.$$

This implies that the put-on-put is A = **\$0.51**.