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<http://www.math.louisville.edu/~ewa/>

Course M seminar: <http://www.math.ilstu.edu/actuary/prepcourses.html>

Course M Casualty/Property Manual: <http://www.neas-seminars.com/registration/>

Practice Problem for exam MFE for the week after 10/27/07.

Company A is a U.S. international company, and Company B is a Japanese local company. Company A is negotiating with Company B to sell its operation in Tokyo to Company B. The deal will be settled in Japanese yen. To avoid a loss at the time when the deal is closed due to a sudden devaluation of yen relative to dollar, Company A has decided to buy a currency option of the European type to hedge that risk.

You are given the following information:

- The deal will be closed in 3 months from now.
- The sale price of the Tokyo operation has been settled at 110 billion Japanese yen.
- The continuously compounded risk-free interest rate in U.S. is 0.035.
- The continuously compounded risk-free interest rate in Japan is 0.015.
- The current exchange rate is 1US dollar = 110 Japanese yen.
- Currency option type: At the money European call option on the yen per dollar exchange rate.
- The logarithm of the yen per dollar exchange rate is an arithmetic Brownian motion with monthly volatility 1.14434%.

Calculate the price in US dollars that company A has to pay for the option.

- (A) less than \$4 million
- (B) at least \$4million but less than \$4.5 million
- (C) at least \$4.5 million but less than \$5 million
- (D) at least \$5million but less than \$5.5 million
- (E) more than \$5.5 million

Solution. (E)

Let $U(t)$ denote the exchange rate of Japanese yen per US dollars at time t .

Because the logarithm of the yen per dollar exchange rate is an arithmetic Brownian motion, $\{U(t)\}$ is a geometric Brownian motion and we can use Black-Sholes formula to price European options.

Company A wants to receive exactly 1 billion US \$ three months from now. The exchange rate now is $U(0) = 110$ yen/dollar. Because Company A is worried about increase in that rate, it will buy a 3-month European call option, whose payoff 3 month from now is:

$$\text{\$billion} \times \text{Max}[U(1/4) \text{ billion} - 110 \text{ billion}, 0] = \text{\$billion} \times \text{Max}[U(1/4) - 110, 0]$$

Since we can use Black-Sholes, the time-0 price of this call option can be computed as follows:

$$\text{\$billion} \times \left[U(0)e^{-r_f T} N(d_1) - Ke^{-rT} N(d_2) \right]; \text{ where}$$

$$d_1 = \frac{\ln(U(0)/K) + \left(r - r_f + \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}} \quad \text{and} \quad d_2 = d_1 - \sigma\sqrt{T}.$$

$$\text{In our case, } U(0) = K = 110, \quad r = 0.015, \quad r_f = 0.035, \quad T = \frac{1}{4}.$$

We know that the monthly volatility is 1.14434%.

$$\text{It means that } \sigma\sqrt{\frac{1}{12}} = 1.14434\% = 0.0114434 \Rightarrow \sigma = 0.05.$$

$$d_1 = \frac{\ln(U(0)/K) + \left(r - r_f + \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}} = \frac{\left(0.015 - 0.035 + \frac{1}{2}0.05^2\right)0.25}{0.05\sqrt{0.25}} = -0.1875. (\text{take } -0.19)$$

$$d_2 = d_1 - \sigma\sqrt{T} = -0.1875 - 0.05\sqrt{0.25} = -0.2125 (\text{take } -0.21)$$

We can compute the time-0 value of the call:

$$\begin{aligned} & \text{billion} \times \left[U(0)e^{-r_f T} N(d_1) - Ke^{-rT} N(d_2) \right] \text{ Yen} = \\ & = 110 \text{ billion} \left[e^{-0.035 \times 0.25} N(-0.19) - e^{-0.015 \times 0.25} N(-0.21) \right] \text{ Yen} = \\ & = 110 \text{ billion} \left[e^{-0.035 \times 0.25} [1 - N(0.19)] - e^{-0.015 \times 0.25} [1 - N(0.21)] \right] \text{ Yen} = \\ & = 110 \text{ billion} \{ (0.9913)(1 - 0.5753) - (0.9963)(1 - 0.5832) \} \text{ Yen} = \\ & = 110 \times 0.005747 \text{ billion Yen} = \text{\$5.75 million.} \end{aligned}$$

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