Drivers are classified as either Preferred or Standard and are reclassified at the end of each year according to a homogeneous Markov process. The probability of a driver who is classified as Preferred at $T = 0$ being classified as Preferred at $T = 1$ is 80%. The probability of a driver who is classified as Standard at $T = 0$ being classified as Standard at $T = 2$ is 44%.

Calculate the probability of a driver classified as Standard at $T = 0$ being classified as Preferred at $T = 1$.

(A) Less than 0.35  
(B) At least 0.35 but less than 0.45  
(C) At least 0.45 but less than 0.55  
(D) At least 0.55 but less than 0.65  
(E) At least 0.65
Solution (CAS exam, Fall 2007, # 36)

The following information is given:

a) $Q_0^{(P,P)} = 0.8$ (and therefore $Q_0^{(P,S)} = 0.2$)

b) $2Q_0^{(S,S)} = 0.44$.

We need to find $Q_0^{(S,P)}$.

By considering all possible ways to go from the state $S$ to the state $S$ in 2 units of time, we have:

$$2Q_0^{(S,S)} = Q_0^{(S,S)} \cdot Q_0^{(S,S)} + Q_0^{(S,P)} \cdot Q_0^{(P,S)}$$

and putting in the data:

$$0.44 = \left(1 - Q_0^{(S,P)}\right)^2 + Q_0^{(S,P)} \cdot (0.2)$$

Denoting $Q_0^{(S,P)}$ by $x$ we have the following quadratic equation:

$$0.44 = (1 - x)^2 + 0.2x \quad \text{or} \quad x^2 - 1.8x + 0.56 = 0$$

which solves for $x = 0.4$ or $x = 1.4$.

Since probability can not be larger than 1, we have $Q_0^{(S,P)} = 0.4$.

ANSWER: B

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