The CAS Insurance Company classifies its auto drivers as Preferred (State #1) or Standard (State #2) starting at time 0 at the start of the first year when they are first insured, with reclassifications occurring at the start of each new policy year. The transition-probability matrices $Q_n$ from the state at time $n$ at the start of year $n+1$ to the state at time $n+1$ are

$$Q_n = \begin{bmatrix} 0.6 & 0.4 \\ 0.3 & 0.7 \end{bmatrix} + \frac{1}{n+1} \begin{bmatrix} 0.2 & -0.2 \\ -0.1 & 0.1 \end{bmatrix}.$$

Given that an insured is classified as Standard at the start of the second year, find the probability that that insured will be Preferred at the start of each of the next three years.

- (A) 0.11
- (B) 0.16
- (C) 0.21
- (D) 0.26
- (E) 0.31
Solution. (Like in Study Note MLC-09-08, Problem No. 252)

We are looking for the following probability:

\[
Q_1^{(2,1)} \times Q_2^{(1,1)} \times Q_3^{(1,1)} = \left(0.3 - \frac{0.1}{2}\right) \left(0.6 + \frac{0.2}{3}\right) \left(0.6 + \frac{0.2}{4}\right) = \\
= (0.25)(0.66667)(0.65) = 0.10833.
\]

Answer: A.