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Course MLC/MFE seminars: http://www.math.ilstu.edu/actuary/prepcourses.html
Course MLC Casualty/Property Manual: http://www.neas-seminars.com/registration/

Practice Problem for exam MLC for the week after 10/06/07.

Insurance losses are a compound Poisson process where:
(i) The approvals of insurance applications arise in accordance with Poisson process at a rate of 2000 a day.
(ii) Each approved application has a 25% chance of being from a smoker and 75% chance of being from a non-smoker.
(iii) The insurance is priced so that the expected loss on each approval is –150.
(iv) The variance of the loss amount is 4000 for a smoker and 7000 on a non-smoker.
Calculate the variance for the total losses on one day's approvals.

A. Less than 60,000,000
B. At least 60,000,000 but less than 61,000,000
C. At least 61,000,000 but less than 62,000,000
D. At least 62,000,000 but less than 63,000,000
E. More than 63,000,000

Solution.
Let S_1 = total losses on one day approval of smokers.
Let S_2 = total losses on one day approval of non-smokers.
Then, $S = S_1 + S_2$ = total losses on one day approvals of any applications.
We can model each losses in the following way;
$S_i = Y_1^i + Y_2^i + \ldots + Y_{N_i}^i$; $i = 1$ or 2, where
N_1 = # of approved smokers in a day,
N_2 = # of approved non-smokers in a day,
$N = N_1 + N_2$ = # of approved applications in a day.
Since N has Poisson distribution with parameter $\lambda = 2000$,
N_1 has Poisson distribution with parameter $\lambda_1 = (2000)(0.25) = 500$ and
N_2 has Poisson distribution with parameter $\lambda_2 = (2000)(0.75) = 1500$.
Each S_i has a compound Poisson distribution.
$\mathrm{Var}(S) = \mathrm{Var}(S_1) + \mathrm{Var}(S_2)$
$\mathrm{Var}(S_i) = \lambda_i E\left[(Y^i)^2\right] = \lambda_i\left(\mathrm{Var}\left[Y^i\right] + E\left[Y^i\right]^2\right)$
$E\left[Y^i\right] = -150$
$\mathrm{Var}\left[Y^1\right] = 4000$, $\mathrm{Var}\left[Y^2\right] = 7000$ and finally,
$\mathrm{Var}\left[S_1\right] = (500)[4000 + (-150)^2] = 13{,}250{,}000$.
$\mathrm{Var}\left[S_2\right] = (1500)[7000 + (-150)^2] = 44{,}250{,}000$.
Thus, $\mathrm{Var}(S) = \mathrm{Var}(S_1) + \mathrm{Var}(S_2) = 57{,}500{,}000$.
Answer A.