MATH 314 April 4, 2003

MID II

Name :
SSN(last four digits) :
Section: 11:30-12:20, MWF
Instructor : Jon-Lark Kim

Cheating will be punished. Show your work logically and write neatly. Just guessing answer won't get any credit. **Write your work within the space given below, which will be possible!**

Problem 1. The set of the first four Laguerre polynomials is given

$$\mathcal{B} = \{1,\ 1-t,\ 2-4t+t^2,\ 6-18t+9t^2-t^3\}.$$

Note that $\mathcal{B}$ forms a basis for $\mathbb{P}_3$.
(a) (10pt) Find the coordinate vector of $2-4t+t^2$ relative to $\mathcal{B}$.

(b) (10pt) Find the coordinate vector of $p(t) = 6-15t+8t^2-t^3$ relative to $\mathcal{B}$.

Problem 2. Assume that matrix A and B are row equivalent.

$$A = \begin{bmatrix} 1 & 1 & -3 & 7 & 9 & -9 \\ 1 & 2 & -4 & 10 & 13 & -12 \\ 1 & -1 & -1 & 1 & 1 & -3 \\ 1 & -3 & 1 & -5 & -7 & 3 \\ 1 & -2 & 0 & 0 & -5 & -4 \end{bmatrix}, B = \begin{bmatrix} 1 & 1 & -3 & 7 & 9 & -9 \\ 0 & 1 & -1 & 3 & 4 & -3 \\ 0 & 0 & 0 & 1 & -1 & -2 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(a) (15pt) Find bases for Col(A), Row(A), and Nul(A).

(b) (5pt) Find also rank$A$ and dimension of Nul(A). Then check whether your answer satisfies the rank theorem.

Problem 3. **(5 pt)** By using the definition of a subspace, prove that for an $m \times n$ matrix A, $\text{Nul}(A)$ is a subspace of $\mathbb{R}^n$.

Problem 4. Let

$$H = \{(a, b, c, d)) : a - 2b - c = 0,\ b - 3d = 0,\ 2c + d = 0\}.$$

(a) **(2pt)** Find a matrix A so that H is the nullspace of A.

(b) **(3pt)** Find a basis for H and the dimension of H.

Problem 5. Let $\mathcal{B} = \{\mathbf{b}_1, \mathbf{b}_2\}$ and $\mathcal{C} = \{\mathbf{c}_1, \mathbf{c}_2\}$ be bases of the vector space $\mathbb{R}^2$. Answer the following.

(a) (10pt) Let $\mathbf{b}_1 = 3\mathbf{c}_1 + 4\mathbf{c}_2$ and $\mathbf{b}_2 = -\mathbf{c}_1 + 2\mathbf{c}_2$. Find $[\mathbf{x}]_{\mathcal{C}}$ for $\mathbf{x} = 4\mathbf{b}_1 - 2\mathbf{b}_2$ by computing $P_{\mathcal{C}\leftarrow\mathcal{B}}$.

(b) (10pt) Let

$$\mathbf{b}_1 = \begin{bmatrix} -1 \\ 8 \end{bmatrix}, \mathbf{b}_2 = \begin{bmatrix} 1 \\ -5 \end{bmatrix}, \mathbf{c}_1 = \begin{bmatrix} 1 \\ 4 \end{bmatrix}, \mathbf{2}_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}.$$

Find $P_{\mathcal{C}\leftarrow\mathcal{B}}$ and $P_{\mathcal{B}\leftarrow\mathcal{C}}$.

Problem 6. Let

$$A = \begin{bmatrix} 1 & 2 & 2 \\ 0 & 2 & 2 \\ 0 & 0 & 3 \end{bmatrix}.$$

(a) (5pt) Find all eigenvalues of A.

(b) (15pt) Find a basis for each corresponding eigenspace of A.

Problem 7 (10 pt). Circle True or False for each statement. Then in either case, justify your answer to get full credit. Each is worth 2 points.

(a) True or False : Since $A\mathbf{0} = \mathbf{0}$ for a square matrix A, $\mathbf{0}$ is an eigenvector of A.

(b) True or False : If $c\mathbf{u} = \mathbf{0}$ for some scalar $c \in \mathbb{R}$ and $\mathbf{u} \in \mathbb{R}^n$, then $\mathbf{u}$ must be $\mathbf{0}$.

(c) True or False : If for a given $m \times n$ matrix A, the equation $A\mathbf{x} = \mathbf{b}$ is consistent for any $\mathbf{b} \in \mathbb{R}^m$, then rank$A$=$n$.

(d) True or False : A single nonzero vector by itself is linearly dependent.

(e) True or False : The dimension of Row(A) can be different from the dimension of Col(A).

From Old Mid I (you can see these from my webpage)

Problem 3. Let

$$A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & -1 \end{bmatrix}$$

(a) (10pt) Find the determinant of A by cofactor expansion and row operation methods. Check whether you got the same result.

True or False Question

(d) True or False : $\det(AB) = \det(A^T B^T)$.

(e) True or False : If two rows of an $n \times n$ matrix A are identical, then $\det(A) = 0$.