New Self-Dual Codes over $GF(4)$ with the Highest Known Minimum Weights(full version)

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Abstract

The purpose of this correspondence is to construct new Hermitian self-dual codes over $GF(4)$ of lengths $22, 24, 26, 32$, and 34 which have the highest known minimum weights. In particular, for length 22, we construct 8 new extremal self-dual $[22, 11, 8]$ codes over $GF(4)$ which do not have a nontrivial automorphism of odd order. The existence of such codes has been left open in [9]. For length 24, we construct 12 new optimal self-dual $[24, 12, 8]$ codes with previously unknown weight enumerators. For other lengths, we construct several self-dual codes with the highest known minimum weights whose weight enumerators were previously unknown.

Index Terms. Hermitian self-dual codes, weight enumerators.

I Introduction

A *linear* $[n, k]$ *code* $\mathcal{C}$ over $GF(4)$ is a k-dimensional vector subspace of $GF(4)^n$, where $GF(4)$ is the Galois field with four elements. In this correspondence, $GF(4) = \{0, 1, 2, 3\}$, where $2 = \omega, 3 = \omega^2 = \overline{\omega}$, and $\overline{\omega} = 1 + \omega$. The *weight* $\mathrm{wt}(\mathbf{c})$ of a codeword $\mathbf{c} \in \mathcal{C}$ is the number of nonzero components of $\mathbf{c}$. The minimum nonzero weight d of all codewords in $\mathcal{C}$ is called the *minimum weight* of $\mathcal{C}$. An $[n, k, d]$ code is an $[n, k]$ code with the minimum weight d. The *Hermitian inner product* is defined as

$$\mathbf{x} \cdot \mathbf{y} = x_1\overline{y_1} + \cdots + x_n\overline{y_n} \in GF(4),$$

for two vectors $\mathbf{x} = (x_1, \cdots, x_n)$ and $\mathbf{y} = (y_1, \cdots, y_n)$. The *dual code* $\mathcal{C}^\perp$ of $\mathcal{C}$ is defined as

$$\mathcal{C}^\perp = \{\mathbf{x} \in GF(4)^n | \mathbf{x} \cdot \mathbf{y} = 0 \text{ for all } \mathbf{y} \in \mathcal{C}\}.$$

If $\mathcal{C} = \mathcal{C}^\perp$, then $\mathcal{C}$ is called a *(Hermitian) self-dual* code. For a self-dual $[n, k, d]$ code over $GF(4)$, it is known [14] that the minimum weight is bounded by $d \le 2[n/6] + 2$. A self-dual $[n, n/2, 2[n/6] + 2]$ code is called *extremal*. A self-dual code of length n is called *optimal* if it has the highest minimum weight of any self-dual codes of that length. An extremal self-dual code is automatically optimal.

Let $\mathcal{M}_n$ be the group of all $n \times n$ monomial matrices over $GF(4)$. Let $\mathrm{Gal}(GF(4))=\{1, \tau\}$ be the Galois group of $GF(4)$ where $\tau : x \to x^2$ is the conjugation map. Let $\mathcal{W}_n$ be the semidirect product of $\mathcal{M}_n$ extended by $\mathrm{Gal}(GF(4))$. So an element $T \in \mathcal{W}_n$ can be written as $T = PD\nu$ where P is a permutation (matrix), D is a diagonal matrix with nonzero diagonal entries, and $\nu \in \{1, \tau\}$. Two codes $\mathcal{C}_1$ and $\mathcal{C}_2$ of length n are called *equivalent* provided there exists an element

$T \in \mathcal{W}_n$ such that $\mathcal{C}_2 = \mathcal{C}_1 T$. The *automorphism group* of the length n code $\mathcal{C}$ is the group $\mathrm{Aut}(\mathcal{C}) = \{T \in \mathcal{W}_n \mid \mathcal{C}T = \mathcal{C}\}$. The *monomial automorphism group* of the length n code $\mathcal{C}$ is the subgroup $\mathrm{MAut}(\mathcal{C})$ of $\mathrm{Aut}(\mathcal{C})$, consisting of the elements from $\mathrm{Aut}(\mathcal{C})$ of the form $PD\nu$ with $\nu = 1$.

Let A_i be the number of codewords of weight i in an $[n,k]$ code $\mathcal{C}$. Then the *(Hamming) weight enumerator* $W(x,y)$ of $\mathcal{C}$ is the homogeneous polynomial

$$W(x,y) = \sum_{i=0}^{n} A_i x^{n-i} y^i.$$

All self-dual codes over $GF(4)$ were classified for lengths $n \le 16$ [3, 13]. The extremal self-dual codes were classified for lengths 18 and 20 [10]. Recently Gulliver [6] gave the first examples of self-dual codes of lengths $26, 32, 34, 36$, and 40 which have the highest known minimum weights by constructing several double circulant self-dual codes over $GF(4)$. Further he showed [6] that there are at least 5 optimal self-dual $[24,12,8]$ codes with distinct weight enumerators and 8 self-dual $[26,13,8]$ codes with distinct weight enumerators. We refer to [6] for the highest known minimum weight d of self-dual codes of lengths $22, 24, 26, 32$, and 34.

The purpose of this correspondence is to construct new Hermitian self-dual codes over $GF(4)$ of lengths $22, 24, 26, 32$, and 34 which have the highest known minimum weights. In particular, for length 22, we construct 8 new extremal self-dual $[22,11,8]$ codes over $GF(4)$ which do not have a nontrivial automorphism of odd order. The existence of such codes has been left open in [9]. For length 24, we construct 12 new optimal self-dual $[24,12,8]$ codes with previously unknown weight enumerators. For other lengths, we construct several self-dual codes with the highest known minimum weights whose weight enumerators were previously unknown. More precisely we construct 41 new self-dual $[26,13,8]$ codes with various weight enumerators, 18 new self-dual $[32,16,10]$ codes with various weight enumerators, and 104 new self-dual $[34,17,10]$ codes with various weight enumerators. To distinguish self-dual codes with the same n, k, and d, we compute weight enumerators of self-dual codes of lengths $22, 24, 26, 32$, and 34 which have the highest known minimum weights. Further we give a building-up construction method (Theorem 1), which is the analogue of the building-up construction in [11].

II Construction methods

We prove the following theorem.(note: there was an error about the definition of $\overline{y_i}$ in the author's published paper IEEE-IT, May 2001 pp 1575-1580. I corrected it here.)

Theorem 1. *Let $G_0 = (L|R) = (l_i|r_i)$ be a generator matrix(may not be in standard form) of a self-dual code $\mathcal{C}_0$ over $GF(4)$ of length $2n$, where l_i and r_i are rows of L and R respectively for $1 \le i \le n$. Let $\mathbf{x} = (x_1, \cdots, x_n, x_{n+1}, \cdots, x_{2n})$ be a vector in $GF(4)^{2n}$ with an odd weight. Suppose that $\overline{y_i} := (x_1, \cdots, x_n, x_{n+1}, \cdots, x_{2n}) \cdot (l_i|r_i)$ for $1 \le i \le n$. Here $\overline{y_i}$ is the conjugate of y_i and $\cdot$ denotes the Hermitian inner product. Then the following matrix*

$$G = \left[\begin{array}{cc|cccccc} 1 & 0 & x_1 & \cdots & x_n & x_{n+1} & \cdots & x_{2n} \\ \hline y_1 & y_1 & & & & & & \\ \vdots & \vdots & & & L & & R & \\ y_n & y_n & & & & & & \end{array}\right]$$

generates a self-dual code $\mathcal{C}$ over $GF(4)$ of length $2n+2$.

Table I: GENERATOR MATRICES OF $C_{20,1}$ AND $C_{20,2}$

$$G(C_{20,1}) = \begin{bmatrix} 10000000001122010201 \\ 01000000001112312130 \\ 00100000001323203112 \\ 00010000001313231201 \\ 00001000001231331011 \\ 00000100001201032202 \\ 00000010001030033231 \\ 00000001000100113322 \\ 00000000100001111111 \\ 00000000010010112233 \end{bmatrix}, \quad G(C_{20,2}) = \begin{bmatrix} 10000000001122233110 \\ 01000000001112312103 \\ 00100000001323202030 \\ 00010000001313013001 \\ 00001000001231330100 \\ 00000100001201210031 \\ 00000010001030210113 \\ 00000001000100113322 \\ 00000000100001111111 \\ 00000000010010112233 \end{bmatrix}$$

Proof. It is easy to see that $\mathcal{C}$ has dimension $n+1$. It remains to show that any two rows of G are orthogonal. Note that the first row of G is orthogonal to itself since it has an even weight. Since any two rows of G excluding the first row of G are orthogonal to each other, it suffices to show that the first row of G is orthogonal to any other rows of G. This is true since for $1 \le i \le n$,

$$\begin{aligned} &(1, 0, x_1, \cdots, x_n, x_{n+1}, \cdots, x_{2n}) \cdot (y_i, y_i, l_i | r_i) \\ &= \overline{y_i} + (x_1, \cdots, x_n, x_{n+1}, \cdots, x_{2n}) \cdot (l_i | r_i) \\ &= \overline{y_i} + \overline{y_i} = 0 \text{ in } GF(4). \end{aligned}$$

This completes the proof. □

It will be convenient if we take G_0 as $G_0 = (I_n|A)$ and $x_{n+1} = \cdots = x_{2n} = 1$ if $n \equiv 0 \pmod 4$, or $x_{n+1} = 0, x_{n+2} = \cdots = x_{2n} = 1$ if $n \equiv 2 \pmod 4$ in the above theorem.

Using Theorem 1 we can prove the following.

Theorem 2. *Any self-dual code $\mathcal{C}$ over $GF(4)$ of length $2n$ with minimum distance $d > 2$ is obtained from some self-dual code $\mathcal{C}_0$ of length $2n-2$ (up to equivalence) by the construction in Theorem 1.*

Proof. The proof of this theorem is exactly the same as [11, Theorem 2]. □

III Good self-dual codes of lengths 22,24,26,32, and 34

A Length 22

The weight enumerator of an extremal self-dual $[22, 11, 8]$ code over $GF(4)$ is given in [13] as follows.

$$\begin{aligned} W_{22}(x,y) &= x^{22} + 990x^{14}y^8 + 18480x^{12}y^{10} + 163548x^{10}y^{12} + 728640x^8y^{14} \\ &\quad + 1533609x^6y^{16} + 1349040x^4y^{18} + 385308x^2y^{20} + 14688y^{22}. \end{aligned}$$

Huffman [8, 9] classified all extremal self-dual $[22, 11, 8]$ codes over $GF(4)$ which have a nontrivial automorphism of odd order. Since we are considering linear codes over $GF(4)$, we note that any self-dual $[22, 11, 8]$ code has a trivial automorphism of order 3, i.e. ωI_{22}, where I_{22} is the identity matrix of size 22. Therefore it has been left open in [9] to construct a self-dual $[22, 11, 8]$ code which does not have a nontrivial automorphism of odd order. To do this it suffices to construct a self-dual $[22, 11, 8]$ code whose full *monomial* automorphism group order is (a power of 2)$\times$3. We construct

Table II: NEW EXTREMAL SELF-DUAL CODES OF LENGTH 22 WITH W_{22}

codes C	vectors $\mathbf{x} = (x_1, \cdots, x_{10}, 0, 1, \cdots, 1)$	$\lvert\mathrm{MAut}(C)\rvert$	using
$C_{22,1}$	1 2 2 1 1 1 1 2 1 1	2×3	$C_{20,1}$
$C_{22,2}$	2 2 3 2 2 1 1 2 1 1	$2^2 \times 3$	$C_{20,1}$
$C_{22,3}$	0 0 3 2 2 2 2 2 1 1	$2^2 \times 3$	$C_{20,1}$
$C_{22,4}$	1 0 0 3 2 2 3 2 2 1	$2^3 \times 3$	$C_{20,1}$
$C_{22,5}$	0 3 1 2 0 1 1 1 1 1	$2^3 \times 3$	$C_{20,1}$
$C_{22,6}$	2 3 2 2 2 1 1 2 2 1	2×3	$C_{20,2}$
$C_{22,7}$	0 3 2 1 0 1 1 2 1 1	$2^2 \times 3$	$C_{20,2}$
$C_{22,8}$	2 1 2 1 2 1 3 3 1 1	$2^6 \times 3$	$C_{20,2}$

such codes by using two self-dual $[20, 10, 8]$ codes $C_{20,1}$ and $C_{20,2}$ which are standard forms of codes in [10, pp. 1616]. See Table I for generator matrices of $C_{20,1}$ and $C_{20,2}$. Using Theorem 1 and $C_{20,1}$ or $C_{20,2}$ we construct several self-dual $[22, 11, 8]$ codes whose full monomial automorphism group orders are (a power of 2)$\times$3. See Table II for such codes. Here vectors in the second column correspond to the first 10 coordinates, the rest being $0, 1, \cdots, 1$(9 1's). The third column computes the order of the monomial automorphism group $\lvert\mathrm{MAut}(C)\rvert$ of the corresponding code C. Even though $C_{22,2}, C_{22,3}$ and $C_{22,7}$ have the same monomial automorphism group order $2^2 \times 3$, they are not in fact equivalent by Magma [1]. Similarly $C_{22,4}$ and $C_{22,5}$ with $\lvert\mathrm{MAut}(C)\rvert = 2^3 \times 3$ and $C_{22,1}$ and $C_{22,6}$ with $\lvert\mathrm{MAut}(C)\rvert = 2 \times 3$ are not equivalent. These $[22, 11, 8]$ codes $\mathcal{C}$ hold 1-designs. More precisely, the nonempty set of codewords in $\mathcal{C}$ of fixed weight i where $i = 8, 10$ holds a 1-design by Assmus-Mattson theorem. We used Magma [1] for all computations here and in other lengths. We summarize our results as follows.

Theorem 3. *There are at least* $38 + 8 = 46$ *inequivalent extremal self-dual* $[22, 11, 8]$ *codes over* $GF(4)$*, eight of which do not have a nontrivial automorphism of odd order.*

B Length 24

The existence of an extremal self-dual $[24, 12, 10]$ code over $GF(4)$ was settled in the negative by Lam and Pless [12]. However the existence of an extremal *additive* self-dual $[24, 12, 10]$ over $GF(4)$ is still open [14]. Now it is natural to construct optimal self-dual $[24, 12, 8]$ codes over $GF(4)$. Using invariant theory([14]), we can compute the weight enumerator of any optimal self-dual $[24, 12, 8]$ code over $GF(4)$ as follows.

$$\begin{aligned} W_{24}(x,y) &= x^{24} + (\beta + 513)x^{16}y^8 + +(14112 - 8\beta)x^{14}y^{10} + (28\beta + 170856)x^{12}y^{12} \\ &\quad +(1118880 - 56\beta)x^{10}y^{14} + (70\beta + 3772467)x^8y^{16} + (6219360 - 56\beta)x^6y^{18} \\ &\quad +(28\beta + 4413528)x^4y^{20} + (1034208 - 8\beta)x^2y^{22} + (\beta + 33291)y^{24}, \end{aligned}$$

where β is an undetermined parameter.

Conway and Pless [2] gave a $[24, 12, 8]$ code with W_{24} for $\beta = 243$. Gulliver [6] obtained such codes which are double circulant for $\beta = 1764, 81, 9$, and 576, proving that there are at least five such $[24, 12, 8]$ codes. By Theorem 1 with shortened $[22, 11, 6]$ codes $S2, S3$ and $S4$(Table III) from codes $C_{24,2}, C_{24,3}$, and $C_{24,4}$ in [6] respectively, we obtain several $[24, 12, 8]$ codes with weight enumerator W_{24} for

$$\beta = 0, 27, 36, 63, 108, 114, 117, 171, 189, 225, 252, \text{ and } 279.$$

Table III: GENERATOR MATRICES OF $S2, S3$, AND S4

$$G(S2) = \begin{bmatrix} 100000000000310311122022 \\ 010000000000231203120 21 \\ 001000000000000100231123 \\ 000100000000300021323223 \\ 000010000000330332320 13 \\ 000001000000333320231 30 \\ 000000100000111331021 20 \\ 000000010000300131103 30 \\ 000000001000203012113 22 \\ 000000000100131322100 0 \\ 000000000013331200020 3 \end{bmatrix}, \quad G(S3) = \begin{bmatrix} 100000000000202130321 22 \\ 010000000000321221313 31 \\ 001000000000221311131 12 \\ 000100000000303220113 23 \\ 000010000000102200011 21 \\ 000001000000203013001 33 \\ 000000100000122312232 22 \\ 000000010000333120223 10 \\ 000000001000130313032 0 \\ 000000000100111113230 1 \\ 000000000012233021320 3 \end{bmatrix},$$

$$G(S4) = \begin{bmatrix} 100000000000221220111 1 \\ 010000000000312213011 1 \\ 001000000000301110301 1 \\ 000100000000300220030 1 \\ 000010000000300313003 0 \\ 000001000000130030030 03 \\ 000000100000220030103 00 \\ 000000010000111030110 30 \\ 000000001000122130111 03 \\ 000000000100221220111 10 \\ 000000000010300301111 1 \end{bmatrix}$$

We enumerate such codes in Table IV. Here vectors in the second column correspond to the first 11 coordinates, the rest being $1, \cdots, 1$(11 1's). The third column shows the β that the corresponding code C takes in the weight enumerator W_{24}. The fourth column computes the order of the monomial automorphism group $|\mathrm{MAut}(C)|$ of the corresponding code C. It seems interesting that $|\mathrm{MAut}(C)|$ is always 3 so that $\mathrm{MAut}(C)$ has only trivial automorphism of order 3, i.e. $\mathrm{MAut}(C) =< \omega I_{24} >$, where I_{24} is the identity matrix of size 24 and $\omega^3 = 1$. We have shown the following.

Theorem 4. *There are at least* $5 + 12 = 17$ *inequivalent optimal self-dual* $[24, 12, 8]$ *codes over* $GF(4)$ *with distinct weight enumerators.*

C Length 26

The existence of an extremal self-dual $[26, 13, 10]$ code is still unknown. The weight enumerator of any self-dual $[26, 13, 8$ or $10]$ code over $GF(4)$ is computed as follows.

$$\begin{aligned} W_{26}(x, y) \;=\; & x^{26} + (\beta + 117)x^{18}y^8 + (10140 - 5\beta)x^{16}y^{10} + (4\beta + 152568)x^{14}y^{12} \\ & +(28\beta + 1372176)x^{12}y^{14} + (6828939 - 98\beta)x^{10}y^{16} + (18014568 + 154\beta)x^8y^{18} \\ & +(23947560 - 140\beta)x^6y^{20} + (13966992 + 76\beta)x^4y^{22} + (2740959 - 23\beta)x^2y^{24} \\ & +(3\beta + 74844)y^{26}, \end{aligned}$$

where β is an undetermined parameter.

Table IV: NEW OPTIMAL SELF-DUAL CODES OF LENGTH 24 WITH W_{24}

codes C	vectors $\mathbf{x} = (x_1, \cdots, x_{11}, 1, \cdots, 1)$	β	$\lvert \mathrm{MAut}(C) \rvert$	using
$C_{24,5}$	0 2 3 3 2 1 2 1 1 1 1	0	3	$S2$
$C_{24,6}$	2 3 2 0 3 2 1 1 1 1 1	27	3	$S2$
$C_{24,7}$	2 2 3 0 3 2 1 1 1 1 1	36	3	$S2$
$C_{24,8}$	3 3 2 3 0 2 1 1 1 1 1	63	3	$S2$
$C_{24,9}$	0 0 0 0 2 0 2 1 1 1 1	117	3	$S2$
$C_{24,10}$	3 2 0 1 3 1 1 1 1 1 1	108	3	$S3$
$C_{24,11}$	0 3 3 0 0 2 3 1 1 1 1	114	3	$S3$
$C_{24,12}$	0 3 3 0 1 3 3 0 1 1 1	225	3	$S3$
$C_{24,13}$	1 3 3 1 1 0 3 1 2 3 1	279	3	$S3$
$C_{24,14}$	3 3 1 3 0 3 3 1 1 1 1	171	3	$S4$
$C_{24,15}$	2 1 3 0 2 3 3 1 1 1 1	189	3	$S4$
$C_{24,16}$	1 1 2 3 0 3 3 2 1 1 1	252	3	$S4$

If there exists a self-dual $[26, 13]$ code with $\beta = -117$, then it is an extremal self-dual $[26, 13, 10]$ code. Gulliver [6] constructed first examples of $[26, 13, 8]$ codes. Each code in [6] corresponds to

$$\beta = 273, 39, 0, 156, 1209, 489, 345, \text{ and } 129.$$

Using Theorem 1 with four $[24, 12, 8]$ codes $C_{24,1}, C_{24,2}, C_{24,3}$, and $C_{24,4}$ in [6], we construct such codes with W_{26} for

$$\begin{aligned} \beta \ = \ & 111, 117, 120, 126, 135, 138, 147, 153, 162, 165, 171, 174, 180, 183, 189, \\ & 192, 198, 201, 207, 210, 216, 219, 228, 237, 243, 255, 261, 264, 282, 291, \\ & 297, 300, 309, 315, 318, 324, 327, 336, 369, 372, \text{ and } 558. \end{aligned}$$

We enumerate such codes in Table VI and VII. Here vectors in the second column correspond to the first 12 coordinates, the rest being $0, 1, \cdots, 1$(11 1's). The third column shows the β that the corresponding code C takes in the weight enumerator W_{26}. We have shown the following.

Theorem 5. *There are at least $8 + 41 = 49$ inequivalent self-dual $[26, 13, 8]$ codes over $GF(4)$ with the highest known minimum weights whose weight enumerators are distinct.*

D Length 32

The existence of an extremal self-dual $[32, 16, 12]$ code is still unknown. The weight enumerator of any self-dual $[32, 16, 10$ or $12]$ code over $GF(4)$ is computed as follows.

$$\begin{aligned} W_{32}(x, y) \ = \ & x^{32} + (\beta + 1968)x^{22}y^{10} + (53928 - 7\beta)x^{20}y^{12} + (15\beta + 1056240)x^{18}y^{14} \\ & + (15\beta + 12050514)x^{16}y^{16} + (84996960 - 150\beta)x^{14}y^{18} \\ & + (378\beta + 366751728)x^{12}y^{20} + (942525792 - 546\beta)x^{10}y^{22} \\ & + (510\beta + 1383471000)x^{8}y^{24} + (1072561392 - 315\beta)x^{6}y^{26} \\ & + (125\beta + 383074920)x^{4}y^{28} + (47561904 - 29\beta)x^{2}y^{30} + (3\beta + 860949)y^{32}, \end{aligned}$$

where β is an undetermined parameter.

If there exists a self-dual $[32,16]$ code with $\beta = -1968$, then it is an extremal self-dual $[32,16,12]$ code. Gulliver [6] constructed the first self-dual $[32,16,10]$ code C_{32} which we denote by $C_{32,1}$. This code corresponds to $\beta = -48$. Using Theorem 1 with C_{30} in [6], which is equivalent to Q_{30} in [13], we construct such codes with W_{32} for

$$\begin{aligned}\beta &= -1098, -1092, -1080, -1074, -1065, -1062, -1056, -1047, -1044, -1038,\\ &\quad -1035, -1029, -1020, -1008, -1002, -993, -984, \text{ and } -966.\end{aligned}$$

We enumerate such codes in Table VIII. Here vectors in the second column correspond to the first 15 coordinates, the rest being $1, \cdots, 1$(15 1's). The third column shows the β that the corresponding code C takes in the weight enumerator W_{32}. We have shown the following.

Theorem 6. *There are at least* $1 + 18 = 19$ *inequivalent self-dual* $[32,16,10]$ *codes over* $GF(4)$ *with the highest known minimum weights whose weight enumerators are distinct.*

E Length 34

The existence of an extremal self-dual $[34,17,12]$ code is still unknown. The weight enumerator of any self-dual $[34,17,10$ or $12]$ code over $GF(4)$ is computed as follows.

$$\begin{aligned}W_{34}(x,y) &= x^{34} + (\beta + 2244)x^{24}y^{10} + (28152 - 4\beta)x^{22}y^{12} + (767448 - 6\beta)x^{20}y^{14}\\ &\quad + (60\beta + 11131362)x^{18}y^{16} + (99245388 - 105\beta)x^{16}y^{18}\\ &\quad + (564922512 - 72\beta)x^{14}y^{20} + (588\beta + 2004131088)x^{12}y^{22}\\ &\quad + (4309732152 - 1128\beta)x^{10}y^{24} + (1215\beta + 5374377756)x^{8}y^{26}\\ &\quad + (3580629624 - 820\beta)x^{6}y^{28} + (346\beta + 1112032152)x^{4}y^{30}\\ &\quad + (120918501 - 84\beta)x^{2}y^{32} + (9\beta + 1950804)y^{34},\end{aligned}$$

where β is an undetermined parameter.

If there exists a self-dual $[34,17]$ code with $\beta = -2244$, then it is an extremal self-dual $[34,17,12]$ code. Gulliver [6] constructed the first $[34,17,10]$ code C_{34} which we denote by $C_{34,1}$. This code corresponds to $\beta = -1122$. Using Theorem 1 with C_{32} in [6], we construct many codes with W_{34} for

$$\begin{aligned}\beta &= -1560, -1536, -1515, -1506, -1500, -1494, -1491, -1488, -1482, -1479,\\ &\quad -1476, -1473, -1470, -1467, -1464, -1461, -1458, -1455, -1452, -1449,\\ &\quad -1446, -1443, -1440, -1437, -1434, -1431, -1428, -1425, -1422, -1419,\\ &\quad -1416, -1413, -1410, -1407, -1404, -1401,\\ &\quad -1398, -1395, -1392, -1389, -1386, -1383, -1380, -1377, -1374, -1371,\\ &\quad -1368, -1365, -1362, -1359, -1356, -1353, -1350, -1347, -1344, -1341,\\ &\quad -1338, -1335, -1332, -1329, -1326, -1323, -1320, -1317, -1314, -1311,\\ &\quad -1308, -1305, -1302,\\ &\quad -1299, -1296, -1293, -1290, -1287, -1284, -1281, -1278, -1275, -1272,\\ &\quad -1269, -1266, -1263, -1260, -1257, -1254, -1251, -1248, -1245, -1242,\\ &\quad -1239, -1236, -1233, -1230, -1227, -1224, -1221, -1215, -1212, -1209,\\ &\quad -1206, -1203, -1197, -1182, \text{ and } -1179,\end{aligned}$$

We enumerate such codes in Table IX, X, and XI. Here vectors in the second column correspond to the first 16 coordinates, the rest being $0, 1, \cdots, 1$(15 1's). The third column shows the β that the corresponding code C takes in the weight enumerator W_{34}. We have shown the following.

Theorem 7. *There are at least* $1 + 104 = 105$ *inequivalent self-dual* $[34,16,10]$ *codes over* $GF(4)$ *with the highest known minimum weights whose weight enumerators are distinct.*

Table V: GOOD ADDITIVE EVEN SELF-DUAL CODES OVER $GF(4)$

n	d_n	No.	reference
22	8	$\geq 38+8$	A
24	8 or 10	≥ 17 or ?	B
26	8 or 10	≥ 49 or ?	C
28	10	≥ 3	[8, 9]
30	12	≥ 1	[13]
32	10 or 12	≥ 19 or ?	D
34	10 or 12	≥ 105 or ?	E

IV Relation to additive even self-dual codes over $GF(4)$

Our results upgrade the known status [14, Table XIV] of additive even self-dual codes over $GF(4)$ with the highest known minimum weights with respect to the trace inner product as well as Hermitian self-dual codes as shown in the previous section. For additive self-dual codes over $GF(4)$ see [4, 5, 7, 14]. We remark that a Hermitian self-dual code over $GF(4)$ is an additive even self-dual code with respect to the trace inner product [4, 5, 14]. In Table V, n denotes the code length, d_n is the highest possible minimum weight for length n, and the third column shows the currently known number of codes. Alphabetical references mean subsections in the section III. We remark that for $n = 22$ in Table V we have to be careful since the 8 codes in Table II have the same weight enumerator W_{22}. We note that in the case of additive self-dual codes with respect to the trace inner product, the equivalence allows each column conjugation while for Hermitian self-dual codes all the columns are conjugated at the same time [4, 5, 7, 14]. However by using the equivalence techinque in [5] we have shown that our 8 new Hermitian self-dual $[22, 11, 8]$ codes in Table II are not equivalent as additive even self-dual codes. Moreover in order to check the equivalence of these 8 codes with the 38 additive even self-dual codes given in [8, 9], we computed the orders of the full automorphism groups of the 8 additive even self-dual codes including each column conjugation. These have (a power of 2)$\times$3. Therefore the 8 codes are not equivalent to the 38 additive even self-dual codes, justifying the entry of $n = 22$ in Table V.

Table VI: NEW SELF-DUAL $[26, 13, 8]$ CODES WITH W_{26}

codes C	vectors $\mathbf{x} = (x_1, \cdots, x_{12}, 0, 1, \cdots, 1)$	β	using
$C_{26,9}$	3 2 1 3 1 1 1 1 1 1 1 2	111	$C_{24,3}$ [6]
$C_{26,10}$	3 1 2 2 1 1 1 1 1 1 1 3	117	$C_{24,3}$
$C_{26,11}$	0 0 1 2 1 1 1 1 1 1 1 2	120	$C_{24,3}$
$C_{26,12}$	1 0 2 2 1 1 1 1 1 1 1 0	126	$C_{24,3}$
$C_{26,13}$	1 1 0 1 1 1 1 1 1 1 1 0	135	$C_{24,3}$
$C_{26,14}$	1 0 0 1 1 1 1 1 1 1 1 1	138	$C_{24,3}$
$C_{26,15}$	0 0 0 2 2 1 1 1 1 1 1 0	147	$C_{24,2}$ [6]
$C_{26,16}$	2 0 0 1 1 1 1 1 1 1 1 2	153	$C_{24,3}$
$C_{26,17}$	2 2 3 2 3 1 1 1 1 1 1 2	162	$C_{24,2}$
$C_{26,18}$	3 2 2 3 2 1 1 1 1 1 1 2	165	$C_{24,2}$
$C_{26,19}$	2 2 2 2 2 1 1 1 1 1 1 2	171	$C_{24,2}$
$C_{26,20}$	0 2 0 2 2 1 1 1 1 1 1 2	174	$C_{24,2}$
$C_{26,21}$	0 3 0 2 2 1 1 1 1 1 1 3	180	$C_{24,2}$
$C_{26,22}$	2 3 3 2 2 1 1 1 1 1 1 2	183	$C_{24,2}$
$C_{26,23}$	1 1 1 1 1 1 1 1 1 1 1 1	189	$C_{24,3}$
$C_{26,24}$	2 2 3 3 2 1 1 1 1 1 1 2	192	$C_{24,2}$
$C_{26,25}$	3 0 3 2 2 1 1 1 1 1 1 0	198	$C_{24,2}$
$C_{26,26}$	2 0 2 2 2 1 1 1 1 1 1 0	201	$C_{24,2}$
$C_{26,27}$	3 0 0 2 3 1 1 1 1 1 1 2	207	$C_{24,2}$
$C_{26,28}$	3 0 0 2 2 1 1 1 1 1 1 3	210	$C_{24,2}$
$C_{26,29}$	2 2 0 2 2 1 1 1 1 1 1 0	216	$C_{24,2}$
$C_{26,30}$	0 0 3 3 2 1 1 1 1 1 1 2	219	$C_{24,2}$
$C_{26,31}$	3 2 3 2 2 1 1 1 1 1 1 2	228	$C_{24,2}$
$C_{26,32}$	0 2 3 3 2 1 1 1 1 1 1 0	237	$C_{24,2}$
$C_{26,33}$	0 0 3 2 3 1 1 1 1 1 1 2	243	$C_{24,2}$
$C_{26,34}$	0 0 2 0 2 1 1 1 1 1 1 0	255	$C_{24,2}$
$C_{26,35}$	0 3 2 0 2 1 1 1 1 1 1 3	261	$C_{24,4}$[6]
$C_{26,36}$	3 0 2 3 2 1 1 1 1 1 1 0	264	$C_{24,2}$
$C_{26,37}$	3 0 3 2 2 1 1 1 1 1 1 0	282	$C_{24,4}$
$C_{26,38}$	3 3 3 2 2 1 1 1 1 1 1 3	291	$C_{24,4}$
$C_{26,39}$	2 3 0 3 2 1 1 1 1 1 1 0	297	$C_{24,4}$
$C_{26,40}$	0 3 3 2 2 1 1 1 1 1 1 0	300	$C_{24,4}$

Table VII: NEW SELF-DUAL $[26, 13, 8]$ CODES WITH W_{26}

codes C	vectors $\mathbf{x} = (x_1, \cdots, x_{12}, 0, 1, \cdots, 1)$	β	using
$C_{26,41}$	3 3 3 3 2 1 1 1 1 1 1 2	309	$C_{24,4}$[6]
$C_{26,42}$	2 0 2 2 2 1 1 1 1 1 1 0	315	$C_{24,4}$
$C_{26,43}$	3 0 0 2 2 1 1 1 1 1 1 3	318	$C_{24,4}$
$C_{26,44}$	3 2 3 2 2 1 1 1 1 1 1 2	324	$C_{24,4}$
$C_{26,45}$	2 3 0 0 2 1 1 1 1 1 1 3	327	$C_{24,4}$
$C_{26,46}$	0 0 3 2 2 1 1 1 1 1 1 3	336	$C_{24,4}$
$C_{26,47}$	0 2 0 2 3 1 1 1 1 1 1 3	369	$C_{24,4}$
$C_{26,48}$	2 0 2 0 2 1 1 1 1 1 1 2	372	$C_{24,4}$
$C_{26,49}$	3 2 2 2 2 1 1 1 1 1 1 3	558	$C_{24,1}$ [6]

Table VIII: NEW SELF-DUAL $[32, 16, 10]$ CODES WITH W_{32}

codes C	vectors $\mathbf{x} = (x_1, \cdots, x_{15}, 1, \cdots, 1)$	β	using
$C_{32,2}$	2 0 2 3 3 1 1 2 1 1 1 1 1 1 2	-1098	C_{30} [6]
$C_{32,3}$	1 3 1 0 3 0 2 1 2 1 1 1 1 1 0	-1092	C_{30}
$C_{32,4}$	3 1 3 2 2 2 2 1 1 1 1 1 1 1 0	-1080	C_{30}
$C_{32,5}$	3 3 2 0 2 2 2 3 1 1 1 1 1 1 3	-1074	C_{30}
$C_{32,6}$	3 2 0 1 0 0 2 1 1 1 1 1 1 1 3	-1065	C_{30}
$C_{32,7}$	3 3 0 0 3 3 1 1 1 1 1 1 1 1 0	-1062	C_{30}
$C_{32,8}$	1 1 2 2 0 2 3 2 1 1 1 1 1 1 3	-1056	C_{30}
$C_{32,9}$	0 3 3 0 1 0 3 1 1 1 1 1 1 1 3	-1047	C_{30}
$C_{32,10}$	0 3 0 3 3 1 3 0 3 1 1 1 1 1 3	-1044	C_{30}
$C_{32,11}$	2 3 0 0 0 2 1 1 1 1 1 1 1 1 3	-1038	C_{30}
$C_{32,12}$	1 1 3 1 3 3 3 2 2 1 1 1 1 1 0	-1035	C_{30}
$C_{32,13}$	0 3 0 3 3 1 3 1 1 1 1 1 1 1 0	-1029	C_{30}
$C_{32,14}$	1 2 0 1 3 0 0 1 2 1 1 1 1 1 3	-1020	C_{30}
$C_{32,15}$	2 0 3 0 0 3 2 2 1 1 1 1 1 1 2	-1008	C_{30}
$C_{32,16}$	0 2 0 3 2 3 3 3 1 1 1 1 1 1 0	-1002	C_{30}
$C_{32,17}$	0 0 0 1 1 1 1 1 0 1 1 1 1 1 0	-993	C_{30}
$C_{32,18}$	2 2 1 0 3 1 2 2 1 1 1 1 1 1 3	-984	C_{30}
$C_{32,19}$	2 3 0 3 3 1 1 2 1 1 1 1 1 1 3	-966	C_{30}

Table IX: NEW SELF-DUAL $[34, 17, 10]$ CODES WITH W_{34}

codes C	vectors $\mathbf{x} = (x_1, \cdots, x_{16}, 0, 1, \cdots, 1)$	β	using
$C_{34,2}$	1 0 0 1 3 0 1 3 0 2 0 0 1 0 2 0	-1560	C_{32} [6]
$C_{34,3}$	1 1 0 2 3 1 0 2 3 1 2 3 3 1 2 1	-1536	C_{32}
$C_{34,4}$	3 1 3 1 1 3 2 2 0 2 1 0 3 3 1 2	-1515	C_{32}
$C_{34,5}$	0 1 2 0 3 3 2 3 0 2 0 0 2 0 0 0	-1506	C_{32}
$C_{34,6}$	1 2 3 1 1 0 2 0 3 0 3 1 1 0 1 1	-1500	C_{32}
$C_{34,7}$	0 0 3 1 3 2 3 3 1 0 0 1 2 1 2 3	-1494	C_{32}
$C_{34,8}$	1 0 1 1 2 3 2 3 1 1 2 1 3 2 2 0	-1491	C_{32}
$C_{34,9}$	1 1 1 3 0 2 1 0 3 0 1 1 0 1 2 1	-1488	C_{32}
$C_{34,10}$	2 1 3 0 1 1 0 1 2 3 0 3 3 1 0 3	-1482	C_{32}
$C_{34,11}$	3 2 1 0 1 3 0 2 2 3 1 1 3 0 0 1	-1479	C_{32}
$C_{34,12}$	0 0 1 0 1 2 2 2 3 2 1 3 1 1 2 0	-1476	C_{32}
$C_{34,13}$	2 1 1 2 1 0 1 0 3 1 0 1 1 0 1 1	-1473	C_{32}
$C_{34,14}$	1 3 2 2 1 0 1 0 0 1 0 2 3 3 0 0	-1470	C_{32}
$C_{34,15}$	0 2 3 0 1 2 1 1 1 0 0 1 1 2 2 3	-1467	C_{32}
$C_{34,16}$	3 1 1 1 0 1 0 0 3 3 0 0 1 0 2 2	-1464	C_{32}
$C_{34,17}$	2 0 1 3 1 2 3 3 1 2 2 3 0 1 3 2	-1461	C_{32}
$C_{34,18}$	3 0 1 3 0 0 0 1 1 3 0 12 2 2 0	-1458	C_{32}
$C_{34,19}$	0 3 1 3 2 0 3 2 3 0 1 2 3 2 0 3	-1455	C_{32}
$C_{34,20}$	0 2 2 2 3 0 3 3 1 0 2 0 0 1 0 1	-1452	C_{32}
$C_{34,21}$	1 0 1 2 1 3 3 2 2 0 1 2 3 1 1 2	-1449	C_{32}
$C_{34,22}$	0 1 1 0 1 1 3 1 1 2 3 3 0 3 2 0	-1446	C_{32}
$C_{34,23}$	0 1 2 0 1 2 2 1 2 2 3 1 1 3 3 2	-1443	C_{32}
$C_{34,24}$	2 1 0 0 1 0 1 0 2 1 1 0 1 0 1 2	-1440	C_{32}
$C_{34,25}$	1 1 2 2 0 1 3 3 0 1 3 1 1 1 1 3	-1437	C_{32}
$C_{34,26}$	1 0 2 2 2 2 2 0 3 2 3 2 0 3 0 2	-1434	C_{32}
$C_{34,27}$	3 3 2 3 2 0 0 1 0 0 2 2 3 2 2 1	-1431	C_{32}
$C_{34,28}$	2 1 1 2 0 3 0 3 3 0 1 0 3 2 1 3	-1428	C_{32}
$C_{34,29}$	3 2 2 2 2 2 0 1 2 3 1 2 1 2 3 0	-1425	C_{32}
$C_{34,30}$	3 3 0 0 0 1 0 3 2 2 0 1 3 0 3 1	-1422	C_{32}
$C_{34,31}$	1 0 3 3 1 3 2 1 2 3 2 0 3 3 3 3	-1419	C_{32}
$C_{34,32}$	2 0 1 2 0 0 1 2 1 3 3 0 3 1 2 2	-1416	C_{32}
$C_{34,33}$	3 2 3 1 2 0 1 0 2 0 0 3 3 3 1 3	-1413	C_{32}
$C_{34,34}$	3 1 0 3 1 2 0 0 2 2 3 0 3 3 0 0	-1410	C_{32}
$C_{34,35}$	3 3 2 2 2 0 0 0 3 0 1 1 0 0 3 1	-1407	C_{32}
$C_{34,36}$	3 0 3 2 2 0 2 3 1 2 3 2 1 3 2 1	-1404	C_{32}
$C_{34,37}$	0 3 1 1 2 1 2 3 0 1 0 3 0 1 1 2	-1401	C_{32}

Table X: NEW SELF-DUAL $[34,17,10]$ CODES WITH W_{34}

codes C	vectors $\mathbf{x}=(x_1,\cdots,x_{16},0,1,\cdots,1)$	β	using
$C_{34,38}$	2 3 1 2 0 0 2 3 0 2 2 3 1 3 1 0	-1398	C_{32} [6]
$C_{34,39}$	0 0 2 1 2 1 1 1 3 0 1 1 0 0 0 1	-1395	C_{32}
$C_{34,40}$	1 3 0 3 3 2 0 3 1 3 1 2 1 1 2 3	-1392	C_{32}
$C_{34,41}$	0 3 0 0 3 1 3 0 2 0 1 0 0 0 3 2	-1389	C_{32}
$C_{34,42}$	2 2 1 2 2 2 1 3 3 0 0 3 2 3 1 2	-1386	C_{32}
$C_{34,43}$	0 0 3 0 2 0 2 1 0 3 0 0 2 1 0 2	-1383	C_{32}
$C_{34,44}$	2 2 1 0 0 1 0 1 3 0 3 3 3 1 3 1	-1380	C_{32}
$C_{34,45}$	3 1 2 0 2 0 0 3 3 2 3 2 0 0 0 2	-1377	C_{32}
$C_{34,46}$	2 0 0 1 1 3 0 1 3 3 3 0 0 1 0 3	-1374	C_{32}
$C_{34,47}$	3 2 2 2 3 1 2 3 0 1 1 1 3 0 0 0	-1371	C_{32}
$C_{34,48}$	3 1 2 1 2 1 2 3 1 2 3 0 0 2 1 2	-1368	C_{32}
$C_{34,49}$	3 2 3 3 0 1 0 3 3 1 2 3 3 1 3 1	-1365	C_{32}
$C_{34,50}$	1 3 3 1 1 3 0 0 1 3 1 2 3 1 2 2	-1362	C_{32}
$C_{34,51}$	1 0 0 3 3 0 1 3 3 3 0 2 2 3 3 2	-1359	C_{32}
$C_{34,52}$	1 1 1 3 2 3 1 1 3 3 3 0 1 2 0 3	-1356	C_{32}
$C_{34,53}$	2 1 3 0 1 2 0 3 1 0 0 2 3 3 0 0	-1353	C_{32}
$C_{34,54}$	2 0 2 3 3 2 3 2 1 2 2 1 3 0 3 3	-1350	C_{32}
$C_{34,55}$	0 1 2 3 2 1 2 0 2 0 3 0 2 3 3 2	-1347	C_{32}
$C_{34,56}$	3 0 0 2 0 3 1 3 0 0 0 2 2 2 0 0	-1344	C_{32}
$C_{34,57}$	0 2 1 1 1 2 0 3 0 2 3 0 1 2 3 1	-1341	C_{32}
$C_{34,58}$	1 1 1 2 1 3 2 3 1 2 1 3 2 3 2 3	-1338	C_{32}
$C_{34,59}$	0 2 0 3 1 1 2 1 2 3 3 1 3 0 1 0	-1335	C_{32}
$C_{34,60}$	1 0 1 0 1 0 0 0 0 1 3 3 1 2 2 2	-1332	C_{32}
$C_{34,61}$	1 1 1 3 2 3 0 0 1 2 1 2 3 0 0 3	-1329	C_{32}
$C_{34,62}$	0 3 3 2 3 3 2 0 0 3 1 1 2 3 1 0	-1326	C_{32}
$C_{34,63}$	3 1 3 3 0 2 2 0 3 2 0 2 1 0 3 3	-1323	C_{32}
$C_{34,64}$	2 1 3 0 1 2 3 0 2 1 1 0 3 2 0 1	-1320	C_{32}
$C_{34,65}$	3 3 1 3 3 0 1 0 3 0 3 3 0 3 1 3	-1317	C_{32}
$C_{34,66}$	0 0 0 0 1 0 0 2 3 2 1 1 3 1 3 1	-1314	C_{32}
$C_{34,67}$	0 0 1 1 1 2 1 3 0 1 2 0 2 1 3 3	-1311	C_{32}
$C_{34,68}$	2 3 2 2 1 2 1 2 1 1 1 2 2 2 0 0	-1308	C_{32}
$C_{34,69}$	0 3 3 2 1 0 3 2 0 0 0 0 1 3 0 0	-1305	C_{32}
$C_{34,70}$	3 2 1 3 3 1 3 0 1 0 3 0 3 0 1 3	-1302	C_{32}

Table XI: NEW SELF-DUAL $[34,17,10]$ CODES WITH W_{34}

codes C	vectors $\mathbf{x}=(x_1,\cdots,x_{16},0,1,\cdots,1)$	β	using
$C_{34,71}$	0 3 2 1 0 2 3 2 3 1 0 3 3 1 0 3	-1299	C_{32} [6]
$C_{34,72}$	2 3 3 2 2 0 2 0 3 1 0 3 2 2 0 2	-1296	C_{32}
$C_{34,73}$	1 0 2 3 3 3 3 1 3 3 3 1 3 1 0 3	-1293	C_{32}
$C_{34,74}$	2 0 1 1 2 1 0 1 3 3 2 0 2 1 2 0	-1290	C_{32}
$C_{34,75}$	1 0 2 0 0 0 2 3 1 3 2 0 3 1 0 1	-1287	C_{32}
$C_{34,76}$	0 1 2 0 3 1 3 2 3 0 2 3 2 0 1 1	-1284	C_{32}
$C_{34,77}$	3 1 3 3 2 0 1 0 2 2 0 1 0 0 2 0	-1281	C_{32}
$C_{34,78}$	0 3 1 1 3 2 2 0 2 1 0 1 1 3 0 2	-1278	C_{32}
$C_{34,79}$	0 2 0 1 1 2 0 0 2 1 0 2 2 0 2 3	-1275	C_{32}
$C_{34,80}$	3 0 2 0 1 1 3 1 2 1 1 3 1 1 1 1	-1272	C_{32}
$C_{34,81}$	1 2 0 1 1 2 1 0 3 2 0 2 2 0 2 1	-1269	C_{32}
$C_{34,82}$	3 0 3 3 0 0 2 0 2 0 0 0 3 3 0 1	-1266	C_{32}
$C_{34,83}$	1 2 0 2 2 3 1 3 0 2 1 1 2 2 1 1	-1263	C_{32}
$C_{34,84}$	1 0 0 3 3 3 2 1 1 0 1 3 2 0 2 1	-1260	C_{32}
$C_{34,85}$	0 2 1 0 3 1 3 3 2 0 2 1 3 3 0 2	-1257	C_{32}
$C_{34,86}$	3 0 1 3 2 2 1 0 3 3 0 0 3 2 2 2	-1254	C_{32}
$C_{34,87}$	3 2 0 2 1 1 1 2 1 0 1 0 1 0 2 3	-1251	C_{32}
$C_{34,88}$	3 0 0 3 2 3 3 1 1 3 2 1 1 2 3 3	-1248	C_{32}
$C_{34,89}$	3 3 1 2 2 1 3 3 0 2 2 2 3 2 0 2	-1245	C_{32}
$C_{34,90}$	1 1 2 1 2 1 1 3 2 3 1 2 3 1 0 0	-1242	C_{32}
$C_{34,91}$	1 2 3 0 3 1 1 3 2 1 0 0 0 3 0 0	-1239	C_{32}
$C_{34,92}$	0 0 1 3 3 1 0 0 1 1 3 0 3 3 0 1	-1236	C_{32}
$C_{34,93}$	2 2 0 3 1 1 3 2 0 0 2 2 0 3 1 3	-1233	C_{32}
$C_{34,94}$	2 0 3 0 1 3 1 3 2 0 1 2 1 0 0 0	-1230	C_{32}
$C_{34,95}$	3 1 1 3 3 1 2 2 3 3 3 1 2 1 3 3	-1227	C_{32}
$C_{34,96}$	1 1 3 3 3 2 3 2 2 3 1 3 1 1 2 1	-1224	C_{32}
$C_{34,97}$	0 3 2 2 2 2 1 0 2 2 3 1 2 1 2 3	-1221	C_{32}
$C_{34,98}$	0 3 1 0 3 1 0 3 1 0 2 0 1 1 0 3	-1215	C_{32}
$C_{34,99}$	3 3 3 0 2 0 1 2 3 0 3 0 2 1 1 3	-1212	C_{32}
$C_{34,100}$	0 3 3 2 0 2 2 0 0 0 3 2 2 1 2 0	-1209	C_{32}
$C_{34,101}$	3 0 1 2 2 1 0 3 2 0 3 1 2 1 0 2	-1206	C_{32}
$C_{34,102}$	2 3 2 0 3 2 0 0 1 2 2 3 3 3 3 0	-1203	C_{32}
$C_{34,103}$	2 0 1 1 2 2 0 0 3 2 0 2 0 0 1 3	-1197	C_{32}
$C_{34,104}$	0 3 3 0 2 1 2 1 0 2 0 3 0 3 3 0	-1182	C_{32}
$C_{34,105}$	2 0 3 1 1 1 3 3 0 0 1 0 1 3 3 1	-1179	C_{32}

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