

QUIZ 1 (Sec 4.4 - 5.2)

Name :

SSN(last four digits) :

Section: 9:30-10:20, MWF

Instructor : Jon-Lark Kim

Cheating will be punished. Show your work logically and write neatly. Just guessing answer won't get any credit. You may use the other side. But all final answers should be on *this* side.

Problem 1. Evaluate the following integrals.

(a) (3 pts)

$$\int_1^2 \frac{x^2 - 3x + 4}{x^2} dx.$$

(b) (3 pts)

$$\int \frac{1}{x \ln \sqrt{x}} dx.$$

Problem 2. (4 pt) Sketch and find the area of the region bounded by the curve $x = 3y$, $x = 2 + y^2$.

QUIZ 2 (5.3)

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Problem 1. Let R be the region bounded by $y = x^2$ and $y = 2 - x^2$.

(a) (1 pts) Sketch R .

(b) (4 pts) Compute the volume of the solid obtained by revolving R with respect to the line $x = 3$.

Problem 2. (5 pt) Let R be the region bounded by $y = x^2$ and $y = 1$. Evaluate the volume of the solid obtained by revolving R with respect to the line $y = 2$.

QUIZ 3 (5.6, 6.4)

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Problem 1. (5 pt) Compute the mass and center of mass of an object with density $\rho(x) = 4 + \frac{x^2}{4}$ kg/m, $-2 \leq x \leq 2$.

Problem 2. (5 pt) The half-life of uranium-235 is about 0.7×10^9 years. If 50 grams are buried at a nuclear waste site, how much will remain after 100 years.

Mid I

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Circle your TA : Jacob Huseby, Aditi Kar, Silvia Saccon

Instruction Cheating will be punished. Show your work logically and write neatly to get partial credit. You may use your calculator but have to show all nontrivial steps. Just guessing answer won't get any credit.

<i>Problems</i>	1	2	3	4	5	<i>Totals</i>
<i>Points</i>	20	20	20	20	20	100
<i>Score</i>						

Problem 1. (20 pts) Evaluate $\int_1^e \frac{1}{x(\ln x+1)^2} dx$ using the integration by substitution.

Problem 2. Let R be the region bounded by the curves $y = \sqrt{x}$ and $y = x^2$.

(a) (10 pts) Sketch R and express its area in terms of one or more integrals. Do not evaluate the integrals.

(b) (10 pts) Write down an integral for the volume of the solid obtained by revolving R about the line $x = 2$. Do not evaluate the integral.

Problem 3. (20 pts) Suppose an object has density $\rho(x) = 3 - \frac{x}{6}$ kg/m, $0 \leq x \leq 6$. Find the center of mass of the object. Also briefly explain why the center of mass should be less than 3 in terms of $\rho(x)$.

Problem 4. (20 pts) Suppose a bacterial culture **triples** in population every 5 hours. If the population is initially 200, find an equation for the population at any time (in hours).

Problem 5. (20 pts) Evaluate either (a) $\int \frac{3x^2}{\sqrt{1-x^6}} dx$ or (b) $\int \frac{3x^2}{1+x^6} dx$ but not both!

Mid I (with solution)

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<i>Problems</i>	1	2	3	4	5	<i>Totals</i>
<i>Points</i>	20	20	20	20	20	100
<i>Score</i>						

Problem 1. (20 pts) Evaluate $\int_1^e \frac{1}{x(\ln x + 1)^2} dx$ using the integration by substitution.

Ans. Let $u = \ln x + 1$.

Then $x = 1$ gives $u = 1$ and $x = e$ gives $u = 2$. Also we have

$$du = \frac{1}{x} dx.$$

Therefore

$$\begin{aligned} \int_1^e \frac{1}{x(\ln x + 1)^2} dx &= \int_1^2 \frac{1}{u^2} du \\ &= -\frac{1}{u} \Big|_1^2 \\ &= \frac{1}{2}. \end{aligned}$$

Problem 2. Let R be the region bounded by the curves $y = \sqrt{x}$ and $y = x^2$.

(a) (10 pts) Sketch R and express its area in terms of one or more integrals. Do not evaluate the integrals.

Ans. I leave the sketch of R to you.

The area of R is given by

$$\int_0^1 (\sqrt{x} - x^2) dx.$$

(b) (10 pts) Write down an integral for the volume of the solid obtained by revolving R about the line $x = 2$. Do not evaluate the integral.

Ans. We apply the volume technique by cylindrical slices. So find the radius = $2 - x$ and height = $\sqrt{x} - x^2$.

$$\therefore \text{the volume} = \int_0^1 2\pi(2 - x)(\sqrt{x} - x^2) dx.$$

Problem 3. (20 pts) Suppose an object has density $\rho(x) = 3 - \frac{x}{6}$ kg/m, $0 \leq x \leq 6$. Find the center of mass of the object. Also briefly explain why the center of mass should be less than 3 in terms of $\rho(x)$.

Ans.

$$\begin{aligned}\text{Mass} &= \int_0^6 \rho(x) \, dx \\ &= \int_0^6 \left(3 - \frac{x}{6}\right) \, dx \\ &= 3x - \frac{1}{12}x^2 \Big|_0^6 \\ &= 18 - 3 = 15.\end{aligned}$$

$$\begin{aligned}\text{First Moment} &= \int_0^6 x\rho(x) \, dx \\ &= \int_0^6 x\left(3 - \frac{x}{6}\right) \, dx \\ &= \int_0^6 \left(3x - \frac{x^2}{6}\right) \, dx \\ &= \frac{3}{2}x^2 - \frac{1}{18}x^3 \Big|_0^6 \\ &= 42.\end{aligned}$$

\therefore c.o.m. is $\frac{42}{15} = 2.8 < 3$.

For the second question, we observe that $\rho(x)$ is a decreasing function of x . So more mass lies in the left of $x = 3$. Therefore we expect that the center of mass of this object is in the left of the half.

Problem 4. (20 pts) Suppose a bacterial culture **triples** in population every 5 hours. If the population is initially 200, find an equation for the population at any time (in hours).

Ans. Start from

$$y(t) = Ae^{rt}$$
$$200e^{rt}$$

Hence

$$600 = 200e^{3r}$$
$$3 = e^{5r}$$
$$r = \frac{\ln 3}{5}$$

Therefore

$$y(t) = 2000e^{\frac{\ln 3}{5}t}.$$

Problem 5. (20 pts) Evaluate either (a) $\int \frac{3x^2}{\sqrt{1-x^6}} dx$ or (b) $\int \frac{3x^2}{1+x^6} dx$ but not both!

Ans. (a) Let $u = x^3$. Then $du = 3x^2 dx$.

Therefore

$$\int \frac{3x^2}{\sqrt{1-x^6}} dx = \int \frac{du}{\sqrt{1-u^2}}$$
$$= \sin^{-1}(u) + C$$
$$= \sin^{-1}(x^3) + C.$$

(b) Let $u = x^3$. Then $du = 3x^2 dx$.

Therefore

$$\int \frac{3x^2}{1+x^6} dx = \int \frac{du}{1+u^2}$$
$$= \tan^{-1}(u) + C$$
$$= \tan^{-1}(x^3) + C.$$