MATH 205: Sample Final Exam Problems

Following are some sample problems taken from MATH 205 final exams over the last few years. There are more problems here than on a final exam.

1. Differentiate the following functions.

   (a) $f(x) = (x^2 + 1)^2$

   (b) $g(t) = \dfrac{\sin t}{1 + \cos t}$

   (c) $h(w) = x^w$

   (d) $x + xy = e^{xy}$ (Find dy/dx.)

   (e) $e(t) = \ln \sin t^2$

   (f) $p(t) = \log_{3t} t^3$

2. If $y = 10^x$, then find $y''(0)$.

3. Find an equation of the tangent line to $y = x^3 - 9x + 1$ at $x = 0$. Use the tangent line to estimate the value of y when x = 0.1.

4. A softball diamond has the shape of a square with sides 60 feet long. If a player is running from second to third at 24 ft/sec, then how fast is her distance from first base changing when she is 20 feet past second base?

5. A field is to be made into the shape of a rectangle. This field is to be further subdivided into three smaller rectangles by placing two fences parallel to one of the sides. If 1000 m of fencing are to be used, then what is the area of the largest such field?

6. Graph $f(x) = \frac{1}{x+2} + x + 1$, showing all important details such as critical points, concavity and asymptotes. On what intervals is this function decreasing?

7. Find the maximum and minimum values of $y = x^4 - 5x^2 + 4$ on the interval $[1, 2]$.

8. True or false?

   (a) $\exp(\ln x) = x$ for every real number x.

   (b) $\ln(\exp(x)) = x$ for every real number x.

   (c) If f is continuous, then f is differentiable.

   (d) If f is differentiable, then f is continuous.

   (e) If $f'(x) > 0$ for all x, then f has no relative maxima.

9. Acceleration due to gravity on the moon is approximately 5.3 ft/sec^2. How long does it take an object to hit the moon if it is dropped from a height of 10 feet?

10. A conical tank with its vertex pointing down has height 4 m and radius 1 m at the top. Oil flows in at the rate of 0.05 m^3/min. When the depth is 2 m, how fast is the level rising? (The volume of a right circular cone of base radius r and height h is $V = \frac{1}{3}\pi r^2 h$.)

11. Evaluate the following limits

    (a) $\displaystyle\lim_{x\to 0} \frac{2x}{\tan 3x}$.

    (b) $\lim_{x\to\infty} e^x/x^4$

    (c) $\lim_{x\to 0} \dfrac{x + \tan x}{\sin x}$

12. Express the following limit as a definite integral on the interval $[1, 7]$.

$$\lim_{n\to\infty} \sum_{i=1}^{n} \left(2x_i^* + 3\sqrt{x_i^*}\right) \frac{6}{n}$$

13. You wish to fence in a rectangular pasture of 30,000 square feet, with one side on a neighbor's lot. If the neighbor agrees to pay for half the cost of the fence that abuts his lot, what dimensions for the fence will minimize your cost?

14. Use Newton's method to estimate the two zeros of the function $f(x) = x^4 + x - 3$. Start with $x_0 = -1$ for the zero on the left and $x_0 = 1$ for the zero on the right.

15. Differentiate each of the following

    (a) $y = x^7 + \sqrt{7}x - \dfrac{1}{\pi + 1}$

    (b) $y = 2\tan^2 x - \sec^2 x$

(c) $y = (x^2 \cot 5x)^{1/3}$

(d) $y = \log_2(x^2/2)$

(e) $y = t \tan^{-1} t - \frac{1}{2} \ln t$

16. Simplify $(\log_4 x)/(\log_2 x)$.

17. On the moon the acceleration of gravity is 1.6 m/sec^2. If a rock is dropped into a crevasse, how fast will it be going just before it hits bottom 30 sec later?

18. Do the following antiderivatives.

(a) $\displaystyle\int \left(8y - \frac{2}{y^{1/4}}\right) dy$

(b) $\displaystyle\int x^{-3}(x+1)\, dx$

(c) $\displaystyle\int (-3\csc^2 x)\, dx$

19. If $f''(x) = -1$, $f'(0) = -5$ and $f(0) = 2$, then find $f(x)$.

20. Given the three functions $F_1(x) = \frac{x^2}{2} \sin x + C$, $F_2(x) = -x \cos x + C$ and $F_3(x) = -x \cos x + \sin x + C$, which is the antiderivative for $f(x) = x \sin x$? Explain your answer.

21. A particle moves on a number line with acceleration $a = 15\sqrt{t} - 3/\sqrt{t}$, subject to the conditions that $ds/dt = 4$ and $s = 0$ when $t = 1$. Find the position function s.

22. Find a value of c satisfying the conclusion of the Mean Value Theorem for the function $f(x) = \sqrt{x-1}$ on the interval $[1, 3]$.

23. Graph the function $y = x^4 - 4x^3 + 10$. Be sure to label critical points and inflection points.