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## INTEGRAL REPRESENTATIONS FOR A CLASS OF OPERATORS ON $L_E^1$

### Abstract

Let  $(X, \mathcal{A}, \mu)$  be a finite measure space,  $E$  a locally convex Hausdorff space,  $L_E^1$  the space of functions  $f : X \rightarrow E$  which are  $\mu$ -integrable by semi-norms,  $P(\mu, E)$  the space of Pettis integrable functions and  $P_1(\mu, E)$  those elements of  $P(\mu, E)$  which are measurable by semi-norms. We prove that a linear continuous mapping  $T : L_E^1 \rightarrow E$  is of the form  $T(f) = \int gfd\mu$  ( $g \in L^\infty$ ) if and only if  $h(T(f)) = 0$  whenever  $h \circ f = 0$  for any  $f \in L_E^1, h \in E'$ . Similar results are proved for  $P(\mu, E)$  and  $P_1(\mu, E)$ .

### References

- [1] Blondia, C., *Integration in locally convex spaces*, Simon Stevin, **55(3)**(1981), 81–102.
- [2] Diestel, J., Uhl, J. J., *Vector Measures*, Amer. Math. Soc. Surveys, Vol. 15, Amer. Math. Soc., 1977.
- [3] Geitz, Robert F., *Pettis integration*, Proc. Amer. Math. Soc. **82** (1981), no. 1, 81–86
- [4] Meziani, Lakhdar, Almezal, Saleh, Waly, Maha Noor, *Integral structure of some bounded operators on  $L_1(\mu, X)$* , Int. J. Math. Anal., **2**(2008), 437–446.

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- [5] Pallaris Ruiz, Antonio J. *The space of Pettis integrable functions with relatively compact range (Spanish)*, Proceedings of the tenth Spanish-Portuguese conference on mathematics, III (Murcia, 1985), 385–391, Univ. Murcia, Murcia, 1985.
- [6] Schaefer, H. H., *Topological Vector spaces*, Springer Verlag 1986.
- [7] Talagrand, M., *Pettis integration and measure theory*, Memoirs Amer. math. Soc., **307**(1984).
- [8] Thomas, G. E., *Integration of functions with values in locally convex Suslin spaces*, Trans. Amer. Math. Soc., **212**(1975), 61–81.