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Jozef Bobok,* KM FSv ČVUT, Thákurova 7, 166 29 Praha 6, Czech Republic. email: bobok@mat.fsv.cvut.cz

Martin Soukenka, KM FSv ČVUT, Thákurova 7, 166 29 Praha 6, Czech Republic or ÚT AV ČR, Dolejškova 1402/5, 182 00 Praha 8, Czech Republic. email: soukenkam@mat.fsv.cvut.cz

IRREDUCIBILITY, INFINITE LEVEL SETS, AND SMALL ENTROPY

Abstract

We investigate continuous piecewise affine interval maps with countably many laps that preserve the Lebesgue measure. In particular, we construct such maps having knot points (a point x where Dini's derivatives satisfy $D^+f(x) = D^-f(x) = \infty$ and $D_+f(x) = D_-f(x) = -\infty$) and estimate their topological entropy. Our main result is: for any $\varepsilon > 0$ we construct a continuous interval map $g = g_\varepsilon$ such that (i) g preserves the Lebesgue measure; (ii) knot points of g are dense in $[0, 1]$ and for a G_δ dense set of z's, the set $g^{-1}(\{z\})$ is infinite; (iii) $h_{\text{top}}(g) \le \log 2 + \varepsilon$.

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