RESEARCH

Real Analysis Exchange
Vol. 36(2), 2010/2011, pp. 449–451

Jozef Bobok,* KM FSv ČVUT, Thákurova 7, 166 29 Praha 6, Czech Republic. email: bobok@mat.fsv.cvut.cz

Martin Soukenka, KM FSv ČVUT, Thákurova 7, 166 29 Praha 6, Czech Republic or ÚT AV ČR, Dolejškova 1402/5, 182 00 Praha 8, Czech Republic. email: soukenkam@mat.fsv.cvut.cz

IRREDUCIBILITY, INFINITE LEVEL SETS, AND SMALL ENTROPY

Abstract

We investigate continuous piecewise affine interval maps with countably many laps that preserve the Lebesgue measure. In particular, we construct such maps having knot points (a point x where Dini's derivatives satisfy $D^+f(x) = D^-f(x) = \infty$ and $D_+f(x) = D_-f(x) = -\infty$) and estimate their topological entropy. Our main result is: for any $\varepsilon > 0$ we construct a continuous interval map $g = g_\varepsilon$ such that (i) g preserves the Lebesgue measure; (ii) knot points of g are dense in $[0, 1]$ and for a G_δ dense set of z's, the set $g^{-1}(\{z\})$ is infinite; (iii) $h_{\mathrm{top}}(g) \le \log 2 + \varepsilon$.

References

[1] L. Alsedá, J. Llibre and M. Misiurewicz, *Combinatorial dynamics and the entropy in dimension one*, Adv. Ser. in Nonlinear Dynamics **5**, World Scientific, Singapore, 1993.

[2] L. Block and W. A. Coppel, *Dynamics in dimension one*, Lecture Notes in Math. **1513**, Springer, Berlin, 1992.

[3] J. Bobok, *On non-differentiable measure preserving functions*, Real Anal. Exchange, **16** (1990-91), 119–129.

Mathematical Reviews subject classification: Primary: 37B40; Secondary: 26A30
Key words: interval map, knot point, Lebesgue measure, topological entropy
Received by the editors January 22, 2011
Communicated by: Zbigniew Nitecki
*The first author was partly supported by the Grant Agency of the Czech Republic contract number 201/09/0854. He also gratefully acknowledges the support of the MYES of the Czech Republic via contract MSM 6840770010.

[4] J. Bobok, *Strictly ergodic patterns and entropy for interval maps*, Acta Math. Univ. Comenianae, Vol. LXXII, **1** (2003), 111–118.

[5] J. Bobok, *The topological entropy versus level sets for interval maps (Part II)*, Studia Math., **166** (2005), 11–27.

[6] J. Bobok and Z. Nitecki, *The topological entropy of m-fold maps*, Ergod. Th. & Dynam. Sys., **25(2)** (2005), 375–401.

[7] J. Bobok and Z. Nitecki, *The topological entropy of m-fold maps on trees*, Ergod. Th. & Dynam. Sys., **27(3)** (2007), 671–701.

[8] W. J. Gorman, *The homeomorphic transformation of c-sets into d-sets*, Proc. Amer. Math. Soc., **17** (1966), 825–830.

[9] A. Katok and B. Hasselblatt, *Introduction to the modern theory of dynamical systems*, Encyclopedia Math. Appl., **54**, Cambridge University Press, Cambridge, 1995.

[10] R. Mañé, *Ergodic theory and differentiable dynamics*, Springer-Verlag, Berlin, 1987.

[11] M. Misiurewicz, *Horseshoes for mappings of the interval*, Bull. Acad. Polon. Sci., Sér. Sci. Math., **27** (1979), 167–169.

[12] R. R. Phelps, *Lectures on Choquet's Theorem*, D. Van Nostrand Company, Inc., Princeton, New Jersey, 1966.

[13] P. Walters, *An Introduction to Ergodic Theory*, Springer-Verlag, New York, 1982.