1. Prove that the square of any odd integer is an odd integer.

2. Prove that $\sqrt{3}$ is an irrational number. Does a similar argument work to show $\sqrt{6}$ is irrational?

3. Where does the proof of Theorem 1.1.1 (see page 1 of the textbook) break down if we try to use it to prove $\sqrt{4}$ is irrational?

4. Prove that $(A \cap B)^c = A^c \cup B^c$.

5. Use the triangle inequality to establish $|a| - |b| \leq |a - b|$.

6. Given a function $f$ and a set $A$, the notation $f(A)$ means $\{f(x) | \forall x \in A\}$.
   (a) Let $f(x) = x^2$. If $A = [0, 2]$ and $B = [1, 4]$, find $f(A)$ and $f(B)$.
   (b) Does $f(A \cap B) = f(A) \cap f(B)$ in this case?
   (c) Does $f(A \cup B) = f(A) \cup f(B)$?

7. For any arbitrary function $g : \mathbb{R} \to \mathbb{R}$, prove that $g(A \cap B) \subseteq g(A) \cap g(B)$ for all sets $A, B \subseteq \mathbb{R}$.

8. Given a function $f : D \to \mathbb{R}$ and a subset $B \subseteq \mathbb{R}$, let us define $f^{-1}(B) = \{x \in D | f(x) \in B\}$.
   (a) Let $f(x) = x^2$. If $A = [0, 4]$ and $B = [-1, 1]$, then find $f^{-1}(A)$ and $f^{-1}(B)$.
   (b) Does $f^{-1}(A \cap B) = f^{-1}(A) \cap f^{-1}(B)$ in this case?
   (c) Does $f^{-1}(A \cup B) = f^{-1}(A) \cup f^{-1}(B)$?

9. For any arbitrary function $g : \mathbb{R} \to \mathbb{R}$, prove that $g^{-1}(A \cap B) = g^{-1}(A) \cap g^{-1}(B)$ for all sets $A, B \subseteq \mathbb{R}$.

10. Let $x_1 = 1$, and for each $n \in \mathbb{N}$ define $x_{n+1} = \frac{1}{2}x_n + 1$.
    (a) Prove that the sequence $(x_1, x_2, x_3, ...)$ is bounded above by 2.
    (b) Using induction, prove that $(x_1, x_2, x_3, ...)$ is an increasing sequence.
11. If a set $A$ contains $n$ elements, prove that the number of different subsets of $A$ is equal to $2^n$.

12. Let $A$ and $B$ be nonempty, bounded above, and satisfy $B \subseteq A$. Prove that $\sup B \leq \sup A$.

13. Compute, without proofs, the suprema and infima of the following sets:
   (a) $\{n \in \mathbb{N} \mid n^2 < 10\}$.
   (b) $\{n/(m + n) \mid m, n \in \mathbb{N}\}$.
   (c) $\{n/(2n + 1) \mid n \in \mathbb{N}\}$.
   (d) $\{n/m \mid m, n \in \mathbb{N} \text{ with } m + n \leq 10\}$.

14. For each $n \in \mathbb{N}$, let $I_n = (0, \frac{1}{n})$, that is, the open interval from 0 to $\frac{1}{n}$.
   (a) Show that $I_1 \supseteq I_2 \supseteq \cdots \supseteq I_n \supseteq \cdots$.
   (b) Prove that $\bigcap_{n=1}^{\infty} I_n = \emptyset$.

15. Given any two real numbers $a < b$, prove that there exists an irrational number $t$ satisfying $a < t < b$. 