1. (5 points) The gravity on the moon is approximately one sixth of Earth's gravity ($-1.6 m/s^2$). A golf ball is hit with an upward velocity of $87 m/s$, how long until it hits the ground?

Solution: Since the gravity is $1.6 m/s^2$ we know that

$$\frac{d}{dt}v(t) = -1.6 m/s^2,$$

thus the velocity

$$v(t) = \int -1.6 m/s^2 dt = -1.6t + v_0 m/s.$$

Since the initial velocity is $87 m/s$, we have that the position

$$p(t) = \int -1.6t + 87 m/s dt = -0.8t^2 + 87t + p_0 m.$$

Since the golf ball starts on the ground, we know $p_0 = 0m$. Thus, the time the golf ball hits the ground is the solution to

$$0m = -0.8t^2 + 87tm.$$

Solving, we get $t = 0, \frac{87}{0.8}s$. The zero solution corresponds to the golf ball first being hit, so the total time in the air is $108.75s$.

2. (5 points) You know that

$$\int_{-1}^{3} f(x)dx = 3 \qquad \int_{1}^{-1} f(x)dx = -2 \qquad \int_{1}^{2} 4g(x)dx = 16 \qquad \int_{3}^{2} g(x)dx = 5.$$

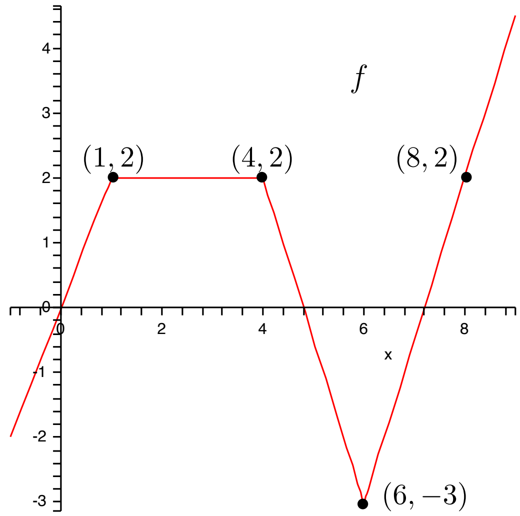
Find $\int_3^1 2f(x) - 3g(x)dx$.

Solution:

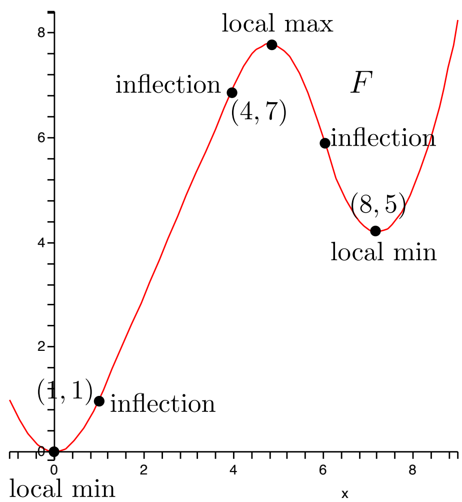
$$\begin{aligned}
\int_3^1 2f(x) - 3g(x)dx &= 2\int_3^1 f(x)dx - 3\int_3^1 g(x)dx \\
&= 2\left(\int_3^{-1} f(x)dx + \int_{-1}^{1} f(x)dx\right) - 3\left(\int_3^2 g(x)dx + \int_2^1 g(x)dx\right) \\
&= 2\left(-\int_{-1}^{3} f(x)dx - \int_{1}^{-1} f(x)dx\right) - 3\left(\int_3^2 g(x)dx + \frac{1}{4}\int_2^1 4g(x)dx\right) \\
&= 2\left(-\int_{-1}^{3} f(x)dx - \int_{1}^{-1} f(x)dx\right) - 3\left(\int_3^2 g(x)dx - \frac{1}{4}\int_1^2 4g(x)dx\right) \\
&= 2\left(-3 - (-2)\right) - 3\left(5 - \frac{1}{4}16\right) \\
&= -5.
\end{aligned}$$

 Name:__________ Points earned:______

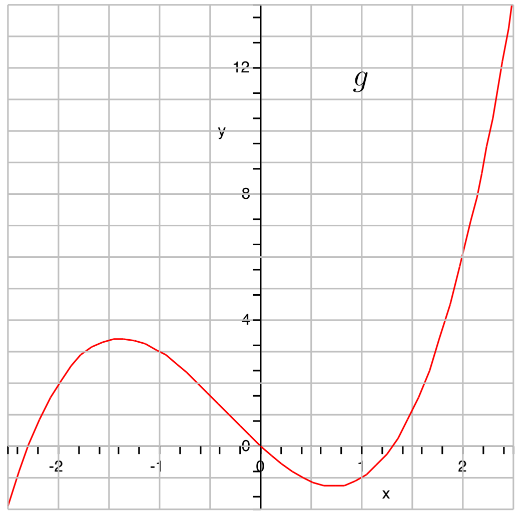
3. (5 points) Below is the graph of $f(x)$. Determine the important points of the antiderivative of f. Sketch the antiderivative of f going through the point $(0,0)$ on the graph provided. Be sure to label any important points and include the values of this antiderivative at $x = 1, 4, 8$.



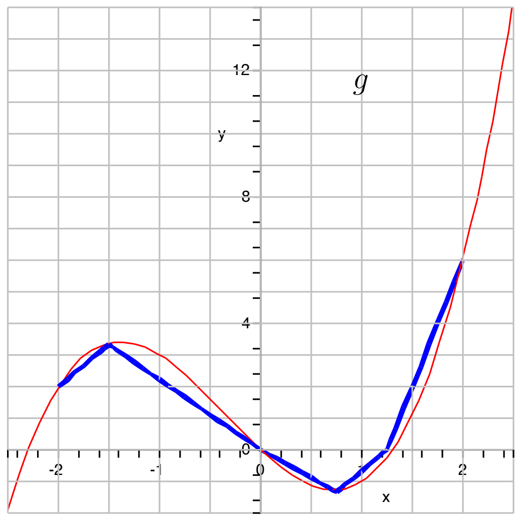
Solution: Since f is 0 at $x = 0, 4.8, 7.2$ these are the local maxima (0 and 7.2) and local minimum. Since f' is undefined at $x = 1, 4, 6$ these are inflection points, with the antiderivative being concave up on $[0,1] \cup [6,8]$ and concave down on $[4,6]$. If F is the antiderivative, then $F(0) = 0$ as given. Then since $\int_0^1 f(x)dx = 1$, $F(1) = 1$. Similarly, $\int_1^4 f(x)dx = 6$ and $\int_4^8 f(x)dx = \frac{2 \cdot 0.8}{2} - \frac{3 \cdot 2.4}{2} + \frac{2 \cdot 0.8}{2} = -2$, so $F(4) = 7$ and $F(8) = 5$.



4. (5 points) Estimate the value of $\int_{-2}^{2} g(x)dx$ given the plot of g given below.



Solution: One possible estimate is given by the blue curve below, which gives an estimate of 5.25. The actual area is $\frac{16}{3}$.

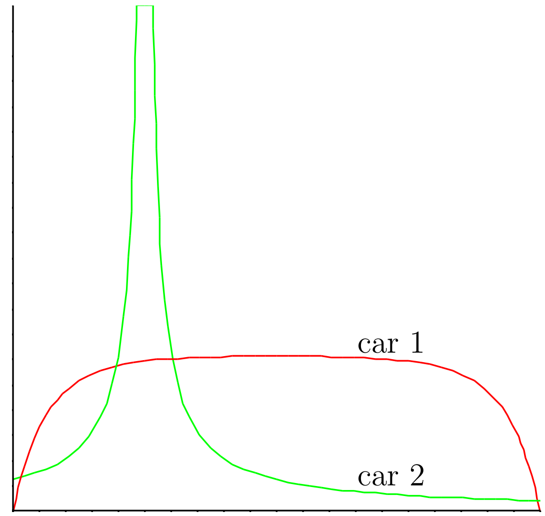


5. (5 points) The velocity of a car in $^{ft}/_{s}$ is given in 2 second increments in the table below. Find two different estimates (each using 5 values) of the distance traveled by the car in the 10 seconds. Is there a relationship between these two values and the actual distance traveled by the car? If so, what?

t	0	2	4	6	8	10
$v(t)$	23	25	33	37	40	45

Solution: The left-hand sum for the position of the car is $23\cdot 2+25\cdot 2+33\cdot 2+37\cdot 2+40\cdot 2 = 158ft$ The right-hand sum for the position of the car is $25\cdot 2+33\cdot 2+37\cdot 2+40\cdot 2+45\cdot 2 = 180ft$. Since we don't know whether the car was constantly increasing speed, we do not know if there is a relationship between these numbers and the distance the car traveled.

6. (5 points) Below are the velocity graphs of two cars. Explain which car you believe has travelled farther at the time the second car reaches its maximum speed and which travels farther overall.



Solution: Looking at the graphs, since the area under the curve for the first car is greater than that for the second car, the second car has travelled further. However, when looking at the area under the curve at the time the second car attained its maximum speed, almost all the area under the velocity curve for the second car is contained in the area under the curve for the first car. I believe that the remaining area is less than the void between the two areas, and thus the first car has gone further when the second car attains its maximum speed.

7. (5 points) Find

$$\int \frac{3}{\cos^2(t)} + t^3 + e^{-3t} dt.$$

Solution:

$$\begin{aligned}\int \frac{3}{\cos^2(t)} + t^3 + e^{-3t} dt &= \int 3\sec^2(t)dt + \int t^3 dt + \int e^{-3t} dt \\ &= 3\tan(t) + \frac{t^4}{4} + \frac{e^{-3t}}{-3} + C.\end{aligned}$$