1. If a, and b, are integers with a<b, when is it true that \(ca>bc\).

*to solve, figure out what c has to be to make the inequality on the right true

➤ This question tests for (chapter 5) representation of integers.

2. Determine all triples of integers (x,y,z) when \(|x|+|y|+|z|=2\)

*To solve, consider absolute value and figure out which 3 numbers will

➤ This question tests on integers covered in chapter 5.

3. For any two integers a and b is it true that \(|a+b| \leq |a|+|b|\)? Why?

*To solve, consider absolute value and consider all integers: positive and negative

➤ This question tests adding integers with absolute value in chapter 5.

4. A 3 x 3 magic square is a square array of 9 numbers such that every row, column, and diagonal sums to the same number. How many times is each position in the square counted in a sum? If the numbers in the square are -7, -5, -3, -2, 0, 2, 3, 5, 7 what should every row, column, and diagonal sum to? Give a magic square for these numbers.

*To solve, find the sum by adding up all the numbers needed for the magic square (-7+-5+-3…). Then divide that number by the number of rows in the square to get the sum. Then use trial and error to complete the square.

➤ This questions tests multiplication of integers in chapter 5.

5. Give an example of a repeating decimal which can be written as a fraction of the form x/9900 and find x.

*To solve, recall the definition of a repeating decimal and the equation to convert the decimal back to the fraction form
This question tests chapter 7 decimals and fractions.

6. Sarah claims that \((\frac{4}{9}) < (\frac{5}{7})\) because \(4<5\) and \(9>7\). Is Sarah right? If not explain why not, if so explain whether this is part of a general pattern.

*To solve, draw the fractions and look for patterns

This question tests Chapter 6 fractions and rational numbers

7. The triangle inequality says that for any numbers \(a, b,\) and \(c\), we get. Explain why this is true.

*to solve, draw it out on a number line to help you explain

This question tests on chapter 5 subtraction of integers with absolute value.

8. If \(a\) and \(b\) are positive integers explain why \(a \times (-b) = - (a \times b)\).

*To solve, use actual numbers and consider the distributive property

This question tests the multiplication of integers in chapter 5.

9. Let \(a\) and \(b\) be positive integers while \(a<b\). Prove that if \(c\) is a negative integer, then \(ac>bc\).

*To solve, it will help to use real numbers

This question tests the relation of integers in chapter 5.

10. Give a list of 5 consecutive numbers and at least one prime factor for each of them. (hint= consecutive: 6,5,4,3,2,1)

*To solve, find 5 numbers in a row that have at least 1 prime factor

This question tests prime factorization and multiplication

11. If \(s\) is proportional to \(t\) and \(s=66.5\) when \(t=9\), what is \(s\) when \(t=14\)
*To solve, set up a proportion and solve for the missing variable s

➤ This question tests proportions and ratios in chapter 7.

12. Use the following numbers to make a magic square: 0.123, 0.246, 0.369, 0.492, 0.615, 0.738, 0.861, 0.984, 1.107

*To solve, find the sum by adding up all the numbers needed for the magic square (-7+-5+-3…). Then divide that number by the number of rows in the square to get the sum. Then use trial and error to complete the square.

➤ This question tests the multiplication of integers.

13. Using each of 1, 2, 3, 4, 5, 6, 7, 8, and 9 once, fill in blanks in this diagram so that the sum of the three-digit numbers formed is 999.

_  _  _

_  _  _

_  _  _

+ _  _  _  = 999

➤ In this question, I am testing over which methods that are covered in chapter one to use in different situations.

14. Study the sequence: 2, 6, 18, 54, 162, … Each number is getting multiplied by 3. Do the sequences a-e.

a. 3, 6, 12, __, __, __

b. 4, __, 16, __, __, __

c. 1, __, __, 216, __, __

d. 2, __, __, __, 1250, __

e. 7, __, __, __, __, 7

Hint: Review methods in chapter one pertaining to patterns.
In this question, I am testing over the problem solving methods that are covered in chapter one.

15. Explain how to illustrate 5 + 2 with

a. The set model of addition

b. The number line (measurement) model of addition

Hint: Review Chapter one methods.

➢ I am testing your ability to be able to visually explain a simple mathematical problem in several different ways.

16. Replace each blank with one of the symbols n, U, c to give a correct statement for general sets A, B, and C. (Hint: U = Union (A combination of two whole sets), n = intersection (Where variables in two or more sets that are the same). Do the problem piece by piece, or break into sections.)

a. A _ C U B

b. A __ (B U C) = (A n B) U(A n C)

➢ In this question, I am testing you on your ability to understand and use union and intersection.

17. 287 as a numeral in each base as indicated

a. Base 5

b. Base 2

c. Base 6

(Hint: Check in the book for how to do base conversions. There is one equation that applies to converting base ten to any other base)

➢ In this question, I am testing on how well you can convert from base ten to other bases.
18. Use the long division algorithm to perform each of these divisions
a. 5/27436
b. 8/39584

Hint: If 8 is not a divisor of 3, bring down the 9 …

In this question I am testing your ability in using the long division algorithm.

19. Use the instructional algorithm for addition to perform the following additions
a. 42 + 54
b. 47 + 35
c. 59 + 63

In this question I am testing your ability to use the instructional algorithm.

20. Rounding to the left most digit, compute approximations to the answers to each of these expressions
a. 657 + 439
b. 657 - 439
c. 657 x 439
d. 1657/23

You will have to do a rough educated guess to get a ballpark figure, you will have to round to the leftmost digit. Remember 5+ rounds up and anything below 4 rounds down

In this question I am testing two different things: your ability to use mental math to get an accurate educated guess, and your ability and understanding on how to round numbers.

21. Write these decimals in expanded exponential form
a. 273.425
b. 0.000354

- In this question I am testing your ability and understanding on how to write numbers in exponential form to standard form.

22. Suppose \( A = 0.2020020002000020000020000002\ldots \) continuing in this way with more 0 between each excessive pair of 2’s. Is this number rational or irrational? Explain briefly.

To be rational the number must be terminating, or repeating.

- In this question I am testing your knowledge of what a ration and irrational number is because to define this number you must have an understanding on what an irrational number is and what a ration number is.

23. Convert 27,896 into base 6. (Hint: assume base ten unless indicated otherwise.)

- The ability of the student to convert from base ten to other base being tested in this question.

24. Determine if 5641 is a composite number and explain your reasoning. (Hint: composite means that a natural number has more than two different factors.)

*Start with finding the prime factorization of the number or by performing long division

- This question is testing the terminology from chapter 3 as well as the ability to factor a number to determine if it is composite or prime.

25. If there is a sale on bananas at the local whole foods store for 23 cents each and the local big name grocery store has bananas on sale for 89 cents per pound. If Jim only wants 5 bananas and the average banana weighs 126 grams. Which store will have the better deal for Jim? (There are about 453.6 grams in one pound.)

*Start by commuting the cost for 5 bananas at each store.

- This question is testing proportional reasoning and conversions.
26. If you invest $250,000 in a share for a new business and the share has an annually compounded interest of 1/10. What percent interest will you earn after 12 years? (Hint: recall the equation on computing credit card rates given in class and apply to this question.)

- This question is testing the ability to compute percentage and understand the relationship between percentages and fractions.

27. Determine if 25,481 is divisible by 9 by performing the divisibility test for 9, and explain your answer. (Hint… _9999 + _999+…..)

- This question is testing the ability to perform divisibility test for 9.

28. Find the GCD and LCM for 806, and 268. (Tip: use which ever method is easiest for you)

- This question tests the ability of finding the GCD and LCM of two numbers using the method of choice.

29. Why are GCD(a,b), and GCD(a,b-a) the same? (Hint: Assume that a is the smaller of the two integers.)

- This questions test the ability to understand the relationship between the GCD of a and b.

30. Using the prime power representation for 76 and 193, find the GCD (76,193) and LCM (76, 193).

*To solve this one must factor the given numbers first, and simplify that factorization into prime powers (i.e 2^3 x 3^2).

- This question tests the ability to use the prime power representation of integers to determine GCD and LCM.
Math Questions
Chapter 1

1. Explain the pattern of Pascal's triangle

   1
   
   1 1
   
   1 2 1
   
   1 3 3 1

2. Find X and Y

<table>
<thead>
<tr>
<th>3</th>
<th>6</th>
<th>9</th>
<th>X</th>
<th>15</th>
</tr>
</thead>
<tbody>
<tr>
<td>18</td>
<td>21</td>
<td>24</td>
<td>27</td>
<td>30</td>
</tr>
<tr>
<td>Y</td>
<td>36</td>
<td>39</td>
<td>42</td>
<td>45</td>
</tr>
</tbody>
</table>

3. Using the chart from question 2 find the value of the shaded box.

   21

4. Find the missing values if the sums of the two adjacent circles must be equivalent to the squares.

   5    a    8
   
   c    b
   
   7
Chapter 2

1. What is 2?

2. How many different ways can the squares be lined up?

3. Is this binary operation commutative? Is there an identity? What is the identity?

4. Using the numbers 1-20, place the integers in the proper location.
Chapter 7

1. What percentage and fraction of the grid is shaded?

2. What percent of the mixture are red jelly beans? What percent is blue?

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Red</td>
<td>16</td>
</tr>
<tr>
<td>Blue</td>
<td>48</td>
</tr>
<tr>
<td>Mix</td>
<td>64</td>
</tr>
</tbody>
</table>
Math Answers

Chapter 1

1. Pascal’s triangle is a figure in which the numbers from the upper tier are added together to form the next row. The ones stay consistently diagonal along the entire triangle while the inner numbers are formed by the sum of their upper diagonal integers.
2. X=12 and Y=33. This can be found by identifying the pattern of n+3 and following it along until you reach the hidden integer.
3. Once again by identifying the pattern of the diagonals being n+18 you can see that the shaded area would be equal to 75. (21+18=39, 39+18=57, 57+18=75)
4. A=3 b=5 c= 2 this can be found by guessing and checking.

Chapter 2

1. We know what 2 is because it is in bijective correspondence with other sets containing 2 things. For example, two coconuts includes a set of one coconut and another coconut making one set of two or 2 coconuts.
2. Six ways:

<table>
<thead>
<tr>
<th></th>
<th>RBO</th>
<th>BOR</th>
<th>OBR</th>
</tr>
</thead>
<tbody>
<tr>
<td>ROB</td>
<td>BRO</td>
<td>ORB</td>
<td></td>
</tr>
</tbody>
</table>

3. It is commutative, the identity is ▲
4.

Chapter 7

1. 1/5 of the graph is shaded, which is 20%
2. 16/64 is ¼ or 25% of the jelly beans making 25% of the jelly beans red. 48/64 is ¾ or 75% making 75% of the jelly beans blue.
Chapter 3

1. Using the long division algorithm, solve for the answer.

\[
\begin{array}{c|cccc}
6 & 780 \\
\hline
\end{array}
\]

\[
\begin{array}{c|cccc}
6 & 780 & 600 & 180 & 120 \\
\hline
10 & 60 & 60 & 60 & 0 \\
\end{array}
\]

Answer: 10 units = 1 strip; 10 strips = 1 mat (100)

2. What calculation does the sequence of sketches represent and explain your answer.

Answer: 10 units = 1 strip; 10 strips = 1 mat (100)
3. Solve the following problem and convert the solution to base SIX.

\[
\begin{array}{c}
\text{143}_\text{FIVE} \\
\times \text{12}_\text{FIVE}
\end{array}
\]

**Answer:** Solve the problem, convert solution to base TEN, and then convert to base SIX.

4. Fill in the missing digits in the addition problem

\[
\begin{array}{c}
2\_37 \\
\_224 \\
+ \_224 \\
\_00\_1
\end{array}
\]

**Answer:**

\[
\begin{array}{c}
2837 \\
7224 \\
+ 7224 \\
10061
\end{array}
\]
Chapter 4

1. Using a factor tree, determine the prime factorization of 728.

Answer: the factor tree may vary, however the prime factorization will not.

\[
\begin{array}{c}
728 \\
\downarrow \\
7 \\
\downarrow \\
2 \\
\downarrow \\
2 \\
\downarrow \\
2 \\
\downarrow \\
2 \\
\downarrow \\
2 \\
\downarrow \\
3 \\
\end{array}
\]

\[2^3 \cdot 7 \cdot 13\]

2. Use a number rod diagram to show that 12 is divisible by 3.

Answer:
3. Determine the GCD of 1029 and 375.

Answer: use the Euclidian algorithm to solve.

\[
\begin{align*}
375 & \div 1029 \\
-756 & \\
279 & \\
375 & \div 279 \\
-279 & \\
910 & \\
279 & \div 910 \\
-279 & \\
910 & \\
87 & \div 910 \\
-87 & \\
9 & \\
\end{align*}
\]

\[
\begin{align*}
9 & \div 87 \\
-81 & \\
6 & \\
9 & \div 6 \\
-6 & \\
3 & \\
6 & \div 3 \\
-2 & \\
2 & \\
\end{align*}
\]

GCD (1029, 375)
= GCD (375, 279)
= GCD (279, 96)
= GCD (96, 87)
= GCD (87, 9)
= GCD (9, 6)
= GCD (6, 3)
= 2
4. Explain why 72561 is divisible by 3.

\[ 72561 \]

\[ 7 \cdot (10000) + 2 \cdot (1000) + 5 \cdot (100) + 6 \cdot (10) + 1 \]

\[ 7 \cdot (9999 + 1) + 2 \cdot (999 + 1) + 5 \cdot (99 + 1) + 6 \cdot (9 + 1) + 1 \]

Because you know that 9 is divisible by 3, you can factor out the 9's in the equation and are left with...

\[ 7 \cdot (1) + 2 \cdot (1) + 5 \cdot (1) + 6 \cdot (1) + 1 \]

Which is equal to...

\[ 7 + 2 + 5 + 6 + 1 = 21 \]

21 is divisible by 3, thus making 72561 divisible by 3 as well.
Chapter 7

1. In order for Anne to buy a new house, she must first make a down payment equivalent to 17% of the total value of her future house. If her down-payment is $35,000; how much does her future house cost?

\[ d = \frac{\%}{100}(x) \]

\[ 35,000 = \frac{17}{100}(x) \]

\[ \frac{35,000}{17} = \frac{17}{17}x \]

\[ $205,882 = x \]

2. Which of the following will give rational answers? Tell why.

\[ \frac{3}{5} \quad \sqrt{7} \quad \frac{5}{8} \quad \frac{789}{999} \quad \pi \]

| \( \frac{3}{5} \) | 3/5=.6 (decimal terminates) |
| \( \sqrt{7} \) | 2.645751311... (decimal neither terminates or repeats) |
| \( \frac{5}{8} \) | 5/8=.625 (decimal terminates) |
| \( \frac{789}{999} \) | 789/999=.789789... (decimal repeats) |
| \( \pi \) | 3.141592654... (decimal neither terminates or repeats) |
MUCK Trial Questions

8 questions

Chapters 5 and 6.

Chapter 5

1. What operations are represented in this diagram?

- In the diagram, blue = positive
- Purple = negative

\[ a + (-2) = 0 \]

* You add up the counters in each bubble than combine the two bubbles

\[ 3 + 2 = 5 \]

* You add up the counters in each bubble than combine the two bubbles

\[ 0 - (-2) = 2 \]

* You add up the counters including the bubble which is zero. Than you add only what is in the bubble which is -2. Then do the subtraction to get 2.
2. What additions and/or subtractions do these number-line diagrams represent?

(a.) 
\[2 - 8 = -6\]

(b.) 
\[-5 + (-4) = -9\]

(c.) 
\[-7 - (-3) = -13\]
3. (a) Jarod's bank account was overdrawn by $16. What was his balance after he deposited the $48 he earned working at a local zoo?

\[ (-16) + 48 = 32 \] in Jarod's bank account.

How you get your answer is by knowing his account was overdrawn so when he deposits money it will put his account back to not being overdrawn.

(b) What arithmetic does this situation illustrate?

\[ (-16) + 48 = 32 \] this is the arithmetic.
4. (a.)

\[ 3 \cdot 3 = 9 \]

(b.)

\[ -5 \cdot 3 = -15 \]

(c.)

\[ -3 \cdot -3 = 9 \]
Chapter 16

1. (a) \[ \frac{3}{6} = \frac{1}{2} \]
   (b) \[ \frac{4}{12} = \frac{1}{3} \]
   (c) \[ \frac{4}{8} = \frac{1}{2} \]
   (d) \[ \frac{3}{8} \]

* How you get the answers is add lines to the shapes to make congruent regions, which enables me to represent the shaded area with fractions.
2. A "2 by 6" piece of lumber is planed from a rough board to a final size of 2 1/2" by 4 1/2". Find the dimensions x and y of the shape created with two such boards.

\[
\text{add. } \quad x = 2 \frac{1}{2} + 4 \frac{1}{2} \\
\quad = 2 + 4 + \frac{1}{2} + \frac{1}{2} \\
\quad = 6 + \frac{1}{2} + \frac{1}{2} \\
\quad = 6 + 1 \\
\quad = 7
\]

- The x dimension of the shape created with the two boards is 7".

- To find the x dimension, you add the width of one piece of lumber to the length of the other piece of lumber.

\[
y = 4 \frac{1}{2} - 2 \frac{1}{2} \\
\quad = \frac{9}{2} - \frac{5}{2} \\
\quad = \frac{4}{2} \\
\quad = 2
\]

- The y dimension of the shape created with the two boards is 2".

- To find the y dimension, you subtract the width of one piece of lumber from the length of the other piece of lumber.
3. Solve this problem found in the Rhind Papyrus: "A quantity and its \( \frac{1}{8} \)th added together become 22. What is the quantity?"

\[
\frac{8}{1} \cdot (x + \frac{1}{8} \cdot x) = 22 \cdot \frac{8}{1}
\]

\[
8x + \frac{1}{8}x = 176
\]

\[
9x = \frac{176}{9}
\]

\[
x = \frac{176}{9} \quad \text{or} \quad 19 \frac{5}{9}
\]

- To get the quantity you just multiply the whole thing by the reciprocal which is \( \frac{8}{1} \).
- This will then give the \( x \)'s on one side, then you divide by the variable with \( x \) which will give you the answer. You will get a big number so you can always make it a mixed number.
4. Give an example of a repeating decimal which can be written as a fraction of the form \( \frac{x}{99000} \) and find \( x \).

\[
100000 - 0.990000 = 1000
\]

\[
\underline{135.78}\]

repeating parts

\[
0.13578 = x
\]

\[
135.78 = 1000x
\]

\[
13578.78 = 100000x
\]

\[
13578.78 - 135.78 = 13443
\]

\[
\frac{13443}{99000}\]

\( x = \frac{13443}{99000} \) multiply each number by the appropriate power of 10.
Chapter 7

1. The square root of 7 is irrational. Explain why \(4 + \sqrt{7}\) and \(2\sqrt{7}\) are also irrational.

\(\sqrt{7}\) is irrational.

Suppose \(4 + \sqrt{7}\) was rational, then \(4 + \sqrt{7} = \frac{a}{b}\).

Thus \(\sqrt{7} = \frac{a}{b} + (-4)\) which is rational.

Because the rational numbers are closed under addition. But we know the square root of 7 is irrational. Which is a contradiction. So \(4 + \sqrt{7}\) is irrational.

Suppose \(2\sqrt{7}\) was rational, then \(2\sqrt{7} = \frac{a}{b}\).

Thus \(\sqrt{7} = \frac{1}{2} \cdot \frac{a}{b}\) which is rational because the rational numbers are closed under multiplication. But we know the square root of 7 is irrational. Which is a contradiction. So \(2\sqrt{7}\) is irrational.
2. Consider the number 1.86527349184527349186527349...

Is this number irrational or rational? Why?

We know that any rational number either repeats or terminates. Since this number 1.86527349... can be represented as a repeating decimal, it is therefore rational.
1. Write the digits 0,2,4,6,8,10,12,14,16 in a 3x3 magic square so that the sums of the rows, columns, and diagonals are all equal.

   Start: Add the numbers up that you have been given. Take the sum of all the numbers and divide it by the size of one side of the box. Your answer will be the total that each row, column, and diagonal will add up to.

2. You are given a deck of cards that have three different attributes; color, shapes, and number of shapes. Each card has either 1, 2, or 3 matching shapes, and the shapes can be circles, squares, or triangles. Each card has shapes that are colored red, blue, or green. Figure out how many cards are in the deck.

   Start: Make a tree that lists out each option for each card. Pick one category that is given and branch off each option giving each possibility for the branches above (i.e. the color red has red circles, red triangles, and red squares. Each red shape can have 1, 2, or 3 shapes on each card.).

3. Add 4312\textsubscript{FIVE} + 1434\textsubscript{FIVE}, and then convert the answer to base TEN.

   Start: Add the two numbers together remembering that you are in base FIVE so the sums of each place value cannot be 5 or greater. Once you have your base FIVE sum, take each number from the sum and multiply it by the base number raised to the corresponding exponential power. The last number of your sum should be raised to the 0 power, and work your way up. The highest exponent power should be one less than the total amount of digits in the number. The sum of the product will give you your answer in base TEN.

4. What is the gcd and lcm of (412,324)?

   Start: Find the prime factorization of 412 and 324. The gcd is the lowest exponential values of the prime factors (0 is included in the lowest of prime factors). The lcm is the highest exponential values from the two prime factors.

5. How many divisors does the number 35,280 have?

   Start: Find the prime factorization of 35,280 (Hint: prime factors will be 2,3,5, and 7). Each exponential value n, of the prime factorization will have divisors from 0 through n. The number of divisors will be a product of all n+1 exponential values.

6. If \( a \geq b \), is \( a > b \) true? Explain.
Start: Look at the difference possibilities of what a and b can be in each case. In order for the statement to be true, it must always be true. If there is even one situation that the expression is false, then the inequality is false.

7. Draw two different diagrams representing the equation $8 + (-3) = 5$.

Start: You can use methods such as a number line and counters. The 8 and the -3 can be represented differently to show the difference between the positive 8 and negative 3 (i.e. black and red counters.). Try to cancel one of the colors out with the same amount of the other color counter, and what is left of the counters is the outcome. The number line will have the two different arrows representing the distance traveled by the 8 from 0, and then the distance from the end point of the first arrow to the final point. The part of the arrow that is not overlapping with the other is your outcome.

8. You are given a bag of marbles. In the bag is are 25 red, 15 blue, and 10 green marbles. What is the probability of drawing a green marble? What is the probability of drawing a red or blue marble?

Start: To find the probability you need to first find the total number of marbles in the bag. You can look at the probability as either a fraction (\# of color marble/total \# of marbles), or as a ratio (\# of color marble:total \# of marbles). Find the simplest term, and the percentage of the fraction or ratio, which can be found through long division.

9. A quantity and its $\frac{1}{6}$ are added together to become 28. What is the quantity?

Start: A quantity equals x, and you want to add $\frac{1}{6}$ of x to the whole quantity x. The sum of x's will equal 28. Now find what x equals.

10. Without performing long division, find the terminating decimal representation of $\frac{245}{40}$.

Start: Find the prime factorization of 245 and 40, and then see what can be canceled from the two. That will leave you with its simplest form. The denominator must have a prime factorization with only 2's and/or 5's in order to be terminating. When the fraction is in its simplest form (which may include making a mixed number) it should make it easier to find the terminating decimal.
EXAM QUESTIONS

1. Mr. Magoo owns a farm of mutated animals. On Mr. Magoo’s farm, all of the donkeys have 6 legs and all of the sheep have only 2. If there are a total of 42 donkeys and sheep and 152 legs, how many of each animal does Mr. Magoo have?

To solve this problem, we can use a system of equations. Let d= the number of donkeys and s= the number of sheep. We know that d+s=152. We know that each donkey has 6 legs and each sheep has 2, so 6d+2s=152. From here, we can use either substitution or elimination to solve the system of equations and figure out how many of each animal is accounted for.

2. The vending machine at the ballpark accepts nickels, dimes and quarters. Finn wants to buy a candy bar that costs 65 cents. If Finn has 5 nickels, 4 dimes, and 2 quarters, how many ways are there for him to pay for his candy bar?

To figure this out, you can make a table of all the possible combinations. Make a column for each of the coins- quarters, dimes and nickels. Start with as many of the highest valued coin as possible then work your way through the smaller ones until you have gone through all possible combinations.

3. Finish labeling the number of elements in the regions in the Venn diagram, where subsets A, B, and C of the universe U meet the following conditions:

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>13</td>
<td>9</td>
<td>19</td>
</tr>
</tbody>
</table>

To solve this, start by filling in as much information as you can from what is given. Then use what you know to figure out the missing information. For example, we know the intersection of A and C must equal 19. We know that part of this intersection (A∩B∩C) is 9. Therefore, we know that the portion of A∩C that does not also intersect with B must be equal to 10, because 19-9=10. By continuing to use the information we know in this way, we will be able to figure out the number of elements in each region.

4. Sheldon the hermit crab is trying to cross a beach that is 1393 miles long to get to the ocean. Each day, Sheldon is able to travel 5 miles towards the ocean. However, each night while the he is sleeping, a
turtle picks Sheldon up and moves him 2 miles backwards. Given these conditions, how many days will it take Sheldon to reach the ocean?

To solve this problem, you must first set up an equation. We will let \( d \) = the number of days it takes Sheldon to reach the ocean. Since Sheldon needs to travel a total of 1393 miles, we know that something \( d \) should yield to 1393. Sheldon can travel 5 miles per day, but he gets taken back 2 miles each night, meaning he travels a total of 5-2= 3 miles every 24 hours. So, we can now say that 3\( d \)=1393 and solve the equation from there.

5. Mrs. Fritz is setting up a new seating arrangement for her students in her classroom. Her class is a rather large lecture with 113 students. She wants to set her desks up into rows with 9 desks in each row. If the answer to a question regarding Mrs. Fritz’s seating arrangement is 12, what is being asked about it? What about if the answer to a question is 5?

This problem would be solved by first dividing the total number of desks needed (113) by the number of desks Mrs. Fritz wants in each row (9) to see how many full rows of 9 Mrs. Fritz would have. When dividing 113 by 9, a quotient of 12 with a remainder of 5. The 12 represents the full number of rows of 9 that can be filled. Since there are then 5 desks left that are not in a row, 5 represents the number of desks left in a 13th uncompleted row.

6. Noah is helping Mr. Adams build an ark. On 5 successive days, Noah works on the ark for 7 hours and 25 minutes, 4 hours and 30 minutes, 5 hours and 20 minutes, 6 hours and 35 minutes, and 5 hours and 25 minutes. How many 8-hour days, how many hours, and how many minutes was all of Noah’s time working on the ark equivalent to?

To start solving this problem, add up the number of hours and the number of minutes. You should get a total of 27 hours and 135 minutes. You can then convert the 135 minutes into 2 hours and 15 minutes by dividing 135 by 60 (the number of minutes in an hour). We can then add the 2 hours and 15 minutes on to the 27 hours to get 29 hours and 15 minutes. Now we must convert 29 hours into 8 hour work days. 29/8=3.625. This means that 3 full 8 hour work days (24 hours) have been worked. We can now see that Noah worked 29-24= 5 more hours after the full 8-hour days. Therefore, Noah worked a total of 8 days, 5 hours, and 15 minutes on Mr. Adam’s ark.

7. Find the least common multiple and the greatest common divisor of 4212 and 88.

To solve this, find the prime factorization of each number. For the greatest common divisor, find the factors that both numbers have in common in their prime factorizations and multiply them together. To find the least common multiple, take the highest power of each of the factors included in the prime factorizations and multiply them together.
8. Complete the magic square using the integers \(-5, -4, -3, -2, -1, 0, 1, 2, \) and 3.

\[
\begin{array}{cc}
2 & \\
& 3 \\
\end{array}
\]

To solve this, first find the sum of all the integers given. The sum in this problem is -9. We now must find the magic sum, which we can represent with x. Since we know that the sum of each row is the same, we can say that \(3x=-9\). Therefore, the magic sum is -3. We can now use the guess and check method and trial and error to find the correct place for each integer so that the sum of each row, column and diagonal is equal to -3.

9. Ashley’s mom is making cupcakes for Ashley’s classmates for her birthday tomorrow. The recipe Ashley’s mom has makes 24 cupcakes. However, there are only 12 students in Ashley’s class. If the following measurements are for making 24 cupcakes, how much of each ingredient should Ashley’s mom use to make only 12?

**Ingredients for 24 cupcakes**

- 2/3 cup butter
- 1 cup sugar
- 4 eggs
- 1 ½ cup flour
- 2 ½ cups milk
- 1 teaspoon vanilla

Since 12 is exactly half of 24, take each of the measurements of the ingredients for 24 cupcakes and divide them by 2 to get the amount Ashley’s mom needs to make only 12.

10. A sidewalk 3.5 feet wide surrounds a field 105.75 feet long by 75.25 feet wide. What is the area of the sidewalk? What is the area of the field?

To find the area of the sidewalk, draw a diagram with the given width of the sidewalk and the length and width of the field. Then, break the sidewalk up into squares and rectangles. Find the dimensions of each of the squares in the corners. Next, find the dimensions of the four rectangles. Now, make an equation for the area of the entire sidewalk using the dimensions just found (area of corners + area of the four rectangles). To find the area of the field, multiply the length by the width (105.75 x 75.25).
Chapter 1:

Maria picks an integer, divided by 2, and then adds 12. She tells you she now has 10. What is her number?

In order to get this number Maria would have to \( \frac{x}{2} + 12 = 10 \to \frac{x}{2} = -2 \). \( x = -4 \) = Marias #s

Chapter 2:

\[ A = \{ n | n \text{ is a cube of a natural number and } 1 \leq n \leq 100 \} \]

\[ B = \{ s | s \text{ is a state of in the United States that borders Mexico} \} \]

Is \( A \neq B \) because they do not contain the same elements but \( A \sim B \) because \( n(A) = |X \) and \( n(B) = 1 \)
Chapter 3:

36 + 75 = 111

= 1 Unit

= 1 strip = 10

= 1 mat = 10

You have 3 strips and six units with 7 strips and 5 units. When added together you get 10 strips and 11 units. The 10 strips will be traded to become one mat and 10 units will be traded to become 1 strip and you have one unit left over giving you 111.

Chapter 4:

List all the divisor factors of 54

54: \{1, 2, 3, 6, 9, 18, 27, 54\} there are 8 divisors (factors) in 54 using the rainbow method.
Chapter 5:
Draw two colored-counter diagrams to represent -3 and 3

... or ........ -3

... or ........ 3

. = Negative Counter

. = Positive Counter

Chapter 6
Express these fractions as mixed numbers

-10 7/9

Step 1: multiply 9 x -10 = -90

Step 2: add -7 + -90 = -97

Step 3: place -97 over 9

-10 7/9 as a fraction is -97/9

Chapter 7
Write the following decimals in expanded form and in expanded notation: 0.20305

0.20305 = 2/10 + 3/1000 + 5/100000

=2x10^-1 + 3x10^-3 + 5x 10^-5
Chapter 4:

Find the prime factorization representation of 24,750

To find the factorization divide 24,750 by factors it is divisible by until you get to a prime number. Therefore 24,750 prime factorization is $24,750 = 2 \times 5 \times 5 \times 5 \times 9 \times 11 = 2^1 \times 3^2 \times 5^2 \times 11^1$

Chapter 1:

A box of apples contains at least 240 and at most 250 apples. How many boxes must be selected to guarantee that 3 boxes contain the same number of apples.

There are 11 holes

240, 241, ……… 249, 250 (both ends are possible 11, not 10 possible numbers of apples.

$2 \times 11 + 1 = 23$ boxes

Chapter 4:

Are 1554, 1999, 805 and 2450 divisible by 30

1554, 1999, 805 and 2450 are not divisible by 30 because none of the numbers are divisible by 2, 5, and 3 at the same time.
1. There is a bird feeder in Mrs. Martins yard. Every night the feeder collects 5 inches of rain, and every day 3 inches of rain evaporate. If the bird feeder holds 23 inches of water, how long will it take for the bird feeder to be filled at night?
   • Concept- The concept this covers is using a diagram to figure out the problem. Also using long division to find out how many days it will take.
   • Answer- To solve this, you need to first do 5-3=2 to find how many inches is held everyday. You can then divide 23 by 2 and you get 11 with a remainder of 1. The birdfeeder will take 11 days to be filled at night.

2. David and Jake are playing a game where they take turns drawing 1, 2, 3, or 4 kites and the person who draws the 25\textsuperscript{th} kite wins. What strategy can Jake use to win, when going first?
   • Concept- The concept this covers is working backwards. You have to do different trials to figure out what it takes to win when Jake goes first.
   • Answer- Try playing with a partner and leaving David with a multiple of 5 each time. Do several times to see if it work every time.

3. Sarah is thinking of a number. If you multiply it by 3 and add 15, you get 42. Could Sarah’s number be 8? Why or why not?
   • Concept- The concept that this covers is setting up an equating and solving. You have to set it up the right way according to how the problem is worded to get the correct answer.
   • Answer- To solve this you can write an equation. Let the number be you are looking for be “x.” If you multiply x by 3 you have 3x. If you add 15 you get 3x+15. And if that equals 42, you can further write, 3x+15=42. Subtract and divide by 3 and you should get the answer 9.

4. If one must follow along the path of the following diagram in the direction of the arrows, how many paths are there from B to C? (May want to redraw so lines are easier to see and not so short).

```
A  ➔  B (diagonal line is from B to E)
↑  ↓
C  ➔  (diagonal line from C goes from C to B)
↑  ↓
E  ➔  D
```

   • Concept- The concept that this covers is considering a special case. You have to look at the diagram and make sure you count the paths carefully and correctly.
   • Answer- There are three different paths.
i. B→E→C 
ii. C→B 
iii. C→A→B 

5. Dana received the grades 78%, 92%, and 92%, on her first three exams. For an A, the average on four exams must be at least a 90%. What does she need to get on her fourth exam to make an A in the class?
   - Concept- The concept behind this is setting up and equation of guessing and checking. Which ever is more preferable to the person working the problem, you can get the answer either way.
   - Answer- If you add the grades and average them you will see what her grade is now. To find what she has to get on the last exam you can guess and check. She needs to get a 100% to get an A in the class.

6. Let the universe be the University of Louisville’s campus. Let C be the set of students at UofL that cheer for the Cards. Let O be the students at UofL that cheer for another team. Let K be the students that cheer for UK. Describe, in words, the set represent by LuK u Ȧ and draw a Venn diagram to represent that set.
   - Concept- The concept that this covers is verifying properties with Venn diagrams. It deals with sets, and you have to make the Venn diagram and shade according to unions, intersections, and compliments.
   - Answer- When you shade it (which I couldn’t figure out how to do on word) L and K should be shaded but none of O should be.

7. Decide which of the following sets are finite.
   i. Every piece of grass in the world 
   ii. Whole numbers divisible by 323,596,913 
   iii. Centimeters on a yard stick 
   - Concept- The concept that this covers is investigating sets. You have to know the meaning of finite and decide if the set is finite or not.
   - Answer- First you need to review what the difference in finite and infinite are. The sets are either finite or infinite. Finite means that the things inside the set can all be counted, and infinite means they go on forever. The answers in order to the three above are: no, yes, and yes.
8. Explain why the GCD of (125,32) = GCD of (32,93). Show more than just a calculation.
   • Concept- The concept that this covers is greatest common factors and being able to figure out why 3 numbers can all have the same greatest common factors.
   • Answer- GCD(125,32) = GCD(32,93) because we can write 125 as 32+93. So if something divided 125 and 32, it also divides 125-93=32. If something divides 125 and 93, it also divides 32+93=125. So if something divides 125 and 32 it divides 93 and 32. So every common divisor of 125 and 32 is a common divisor of 93 and 32. Since the common divisors of 125 and 32 are the same as the common divisors of 32 and 93, their GCD’s have to be the same.

9. A 3x3 magic square is a square array of 9 numbers such that every row, column, and diagonal sums to the same number. If the numbers in the square are -5, -4, -3, -2, -1, 0, 1, 2, and 3, what should every row, column, and diagonal sum to? Draw the magic square and fill it in.

```
2   -5   0
-3   -1   1
-2    3   -4
```

• Concept- The concept that this covers is critical thinking, guessing and checking, and setting up an equation to solve. You have to solve an equation to find out how much each row, column, and diagonal needs to add up too, and then guess and check and do mental math to figure out where the numbers go.

• Answer- Every row, column, and diagonal should sum to -3. You can figure this out by adding all the numbers that you are supposed to use and then divide them by three. They all add up to -9 and -9 ÷ 3 = -3. To figure out where the numbers go in the square you can guess and check until all rows, columns, and diagonal add up to -3. The magic square should end up looking like this.

10. Find the GCD and LCM of 250 and 138.
   • Concept- The concept that this covers is factors and multiples. You have to be able to find the factors of numbers and also the multiples to get the GCD and the LCM.
   • Answer- To find the GCD, list the factors of each of the two numbers. The factors of 250 are 1, 2, 5, 10, 25, 50, 125, and 250. The factors of 138 are 1, 2, 3, 6, 23, 46, 69, and 138. Their greatest common factor, as you can see, is two. There are many different ways to go about finding the LCM. If you multiply the two numbers together you get 250x138=34,500. If you
divide this by the GCD (2) you will get the LCM. 34,500/2= 17,250.
GCD(250,138)=2 and LCM(250, 138)=17250.

11. Find the prime factorization for 325 and by doing so tell how many divisors it has, list them.
• Concept- The concept of this is prime factorization and factor trees. You have to know how to find the prime factorization of a number and to find the amount of divisor you have to be able to set up the equation and solve it. You get the numbers for the equation from the prime factorization.
• Answer- To find the prime factorization of a number you can do a factor tree. When you do a factor tree for the number 325 you get 5, 5, and 13. Therefore the prime factorization is $5^2 \times 13^1$. To tell how many divisors this number has, you use the prime factorization. You multiply the exponent+1 by the exponent+1 for each number in the prime factorization. So there are $(2+1)(1+1)=6$ divisors of 325.

12. Shuffle and ordinary 52-card deck and 26 to you and 26 to a partner. Let the number of red and black cards in your hand be denoted by r and b. let R and B denote the number of red and black cards in your partners hand. Then form the fractions r/b and B/R. Explain what is happening.
• Concept- The concept behind this problem is mathematical reasoning and being able to do different trials and explain the reasoning of your results.
• Answer- The deck has 52 cards divided equally between 2 people. 26 cards were dealt to each person and there are only 2 colors in the deck, therefore only 2 colors dealt to each person. Since only 2 colors were dealt, the number of red dealt to you would be the same number of black dealt to your partner. Therefore, the number of black cards for you would be the same number of red that your partner got.

13. Write 0.546 in expanded form and in expanded exponential form.
• Concept- The concept of this problem is decimals and fractions. You have to know how to change a decimal to a fraction and into expanded exponential form.
• Answer- To write the number 0.546 in expanded form, begin by identifying the place value of each digit. So $5=\frac{1}{10}$, $4=\frac{1}{100}$, $6=\frac{1}{1000}$. Thus the expanded form is $5 \times \frac{1}{10} + 4 \times \frac{1}{100} + 6 \times \frac{1}{1000}$. To write 0.546 in expanded exponential form count the zeros in each place value. It should look like this: $5 \times 10^{-1} + 4 \times 10^{-2} + 6 \times 10^{-3}$.

14. Change 1.45 to fraction form.
• Concept- The concept of this problem is to be able to change a decimal to a fraction. Set up the equation to do so, and subtract.
• Answer- To do this, you need to choose and appropriate power of 10 and subtract the repeating numerals. Let $r=1.4\overline{5}$, since the two digits repeat, multiply by 100. $100r=145.\overline{45}$ then you subtract.
100r = 145.45
- r = 1.454545

\[
\begin{align*}
99r &= 144 \\
r &= \frac{144}{99}
\end{align*}
\]

The repeating decimal part cancels out. Divide both sides by 99 and the answer, simplified, is \(\frac{16}{11}\).

15. It takes Amy 10 minutes to walk to her friend’s house. Assume that the same rate of speed is maintained for the entire distance.
   i. How long, in minutes, will it take Amy to walk halfway to her friend’s house?
   ii. If Amy is halfway to her friend’s house, how long, in minutes, will it take her to walk half of the remaining distance to her friend’s house?
   iii. If Amy walks halfway to her friend’s house and then half of the remaining distance, how long, in minutes, will it take her to walk half the remaining distance?
   iv. What must the sum of the times be in parts i, ii, and iii be?

- **Concept**- The concept of this problem is fractions and dividing. You have to set up fractions, multiple them, and divide them to get the answer.
- **Answer**- For the first part, the answer is 5 minutes. You get this by multiplying \(\frac{1}{2}\) by 10. The answer to the seconds part is 2.5 minutes because \(\frac{1}{2} \times 5 = \frac{5}{2}\), or 2.5 as a decimal. The answer to the third question is 10/8 minutes, you can get this by adding 10/2 and 5/2 and subtracting that from 10 to see how much more time it will take. And the answer to the fourth part is 10 minutes, because all together, it takes 10 minutes to talk to her friend’s house.

16. Perform the additions and subtractions by hand.
   i. \(37.152 + 594.7\)
   ii. \(594.7 - 37.152\)
   iii. \(0.072 + 1.48\)
   iv. \(0.072 - 1.48\)

- **Concept**- The concept of this is adding decimals. Remembering how to line the decimals up and add them.
- **Answer**- When adding decimals, the decimal points should always add up. For the first one, the answer is 631.852. For the second one, the answer is 557.548. For the third one, the answer is 1.552. For the fourth one, the answer is 1.408.

17. Sami bought 5 pairs of socks as Christmas presents for 8 of her friends. If the gloves cost $2.15 a pair, how much did she spend?
• Concept- The concept of this problem is being able to set up an equation and multiply decimals. You have to remember how to line up and where to put the decimal when multiplying.

• Answer- First you need to translate the problem into an equation.

\[
\begin{align*}
2.15 \text{ (the cost of socks per pair)} \\
\times \quad 8 \text{ (number of friends)} \\
\hline
17.20
\end{align*}
\]

Thus, it costs $17.20, all together, to buy each of her 8 friends a pair of socks.

18. A picture frame 4.25 inches wide surrounds a picture that is 20.5 inches wide by 25.75 inches high.
   i. What is the area of the picture frame?
   ii. What is the area of the picture?

• Concept- The concept of this is being able to see what it is asking, setting up an equation, and solving it to find the area.

• Answer- For the first part of the problem, you can find the area by setting up this equation and solving it.

\[
(4) \times (4.25^2)+(2) \times (4.25)\times(20.5)+(2)\times(4.25)\times(25.75)
= 72.25+174.25+218.875
= 465.375 \text{ inches}
\]

For the second part of the problem, you can find that area of the picture by doing width \(\times\) height. So, \(20.5 \times 25.74 = 527.875 \text{ inches}\).

The area of the picture frame is 465.375 inches and the area of the picture is 527.875 inches.

19. Find the amount of 34% of 240.

• Concept- The concept of this is percent’s. You have to know how to change a percent to a decimal, and solve an equation that will give you the percent of a number.

• Answers- The first step is to translate the problem. \(X=34\% \times 240\). Then convert 34\% into decimal notation, so, \(X=0.34 \times 240\). When you multiply, you get 81.6. Thus, 34\% of 240 is 81.6.

20. The mortgage company requires a 15\% down payment on houses it finances. If the Smith’s bought a house for $150,000, how much did they have to pay for the down payment?

• Concept- The concept of this is to be able to set up an equation, change a percent to a decimal, and solve the equation.

• Answer- You first need to set up an equation. \(D=150,000 \times .13\). When you multiply, \(D=19,500\). Therefore, the Smith’s down payment on their $150,000 house is $19,500.

21. If \(a \geq b\) is \(a > b\)? If so, explain how. If not, give a counterexample.
• Concept- The concept of this is integers and being able to think critically and prove a case true or false.
• Answer- No, this statement is not necessarily true. If \( a \geq b \), that means that \( a \) and \( b \) could be equal, making the second statement not necessarily true. A counter example would be if \( a=3 \) and \( b=3 \). \( 3 \geq 3 \), but \( 3 \) is not \( > 3 \).

22. Perform the addition and subtraction of the two problems, being careful to not leave more than 59 seconds/ minutes in your answer.
   i. 5hours, 27minutes, 52seconds
      - 2hours, 40minutes, 30seconds
      ______________
   ii. 3hours, 29minutes, 45seconds
      + 7hours, 42minutes, 30seconds
      ______________

• Concept- The concept of this is place value notation. It isn’t like normal addition and subtraction, because it is time. So you have to remember that you can’t have more than 60 seconds or minutes.
• Answer- The answer to the first one is 2 hours, 47 minutes, and 22 seconds. You get this answer by first subtracting the seconds. Since you cannot subtract 40 from 27, you borrow from the hour’s column, and change the 5 hours to 4 and add 60 minutes to the 27. This makes 87-40 leaving you with 47 minutes. And then 4-2=2 hours.
   For the second problem the answer is 11 hours, 12 minutes, and 15 seconds. When you add 45 and 30, you get 75. Since there are only 60 seconds in a minute, you take away 60 from the 75 and add 1 minute (60 seconds) to the 29. So then you do 30+42, which is 72, which is more than enough. So when you add it all up in the end you get 11 hours, 12 minutes, and 15 seconds.

23. Fill in the blanks and explain how you got your answer.
   i. 31 62 seven
      + _ _2 seven
      ___________
      402 seven
   • Concept- The concept behind this problem is positional notation. You have to know how much each position is worth and be able to add in different bases.
   • Answer- The problem should look like 3162+532=4020. You can figure this out by first doing 2+2=4. Then you have 6+?=2. Since it is base 7, it would be 12, with the blank being 2 and carrying the one. So then 2+? has to =0. 7 is 10 in base 7, so the blank will be a 5, since 5+2=7. Then you carry the 1 and have 3+1=4 and get the final answer.
24. List the properties of whole number addition and give one example of each.
   - **Concept** - The concept behind this is the addition theorem. You have to know what properties are true to whole numbers when adding them.
   - **Answer** - The properties of whole number addition are closure, commutative, associative, and additive identity. Closure - a unique whole number (a+b=c). Commutative - order does not matter (a+b=b+a). Associative - grouping does not matter. [a+(b+c)=(a+b)+c]. Additive identity property of zero. (a+0=a)

25. A 2X4 piece of lumber is planned from a rough board to a final size of 1 ½X2 ½. Find the dimensions x and y of the shape created with two such boards.

   ![Diagram of lumber dimensions]

   - **Concept** - The concept of this is being able to set up equations and solve for the dimensions. Being able to think critically and know what the question is asking.
   - **Answer** - To find the dimensions of x and y you need to set up an equation for x and y. They should look like so:
     
     \[ \begin{align*}
     X &= 2 \frac{1}{2} + 1 \frac{1}{2} \text{ so } x=4 \\
     Y &= 2 \frac{1}{2} - 1 \frac{1}{2} \text{ so } y =1
     \end{align*} \]
     
     The dimension of x would be the length of board number 2 (2 ½ ) plus the width of board number 1 (1 ½ ). Thus the dimension of x would be 4. The dimension of y would be the length of boards number 2 (2 ½ ) minus the width of board number 1 (1 ½ ). Thus, the dimension of y would be 1.

26. If \( ab < ac \) is \( b < c \). When is this true and when is this false?
   - **Concept** - The concept of this is integers and being able to think critically and prove a case true or false.
   - **Answer** - This statement is true when a is positive. This statement is false if a is negative. Sub numbers in for a, b, and c and you can see why.
27. \( F_1=1, F_2=1, F_3=2, [F_{n+1}=F_n+F_{n-1}] \). The first 12 Fibonacci numbers are 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, and 144. Determine which numbers are odd and explain why they are.

- Concept- The concept behind this is the Fibonacci numbers and understanding a different number system, how it works, and why it works.
- Answer- The odd number in the first 12 Fibonacci numbers are 1, 1, 2, 3, 5, 13, 21, 55, and 89. Every other 2 numbers are odd. This is because when you add an odd and even number you get an odd number. 1 (odd) plus 2 (even) = 3 (odd). 3 (odd) + 3 (odd) = 5 (odd). Then there is an even number (5+3=8) because 2 odds added together give you an even number. And the pattern continues.

28. Determine the base used in each case and explain how you figured this out.
   i. \( \underline{231} \)
      +\( \underline{414} \)
      \underline{1200} 
   ii. \( \underline{344} \)
      +\( \underline{143} \)
      \underline{531} 
   iii. \( \underline{1020} \)
      -\( \underline{203} \)
      \underline{312} 

- Concept- The concept to this is positional notation. You have to know the position and value of the numbers in different bases to find out what the base of the problems are.
- Answer- One way to find out the bases are to guess and check. The first one is base 5 because when you add 4+1 and get 0, and carry the one above the three and get 0 again, and carry the one above the 2, you get 7, which is 12 in base 5. The second one is in base 6. There are no numbers larger than 5 and when you add 4 and 3 you get 7, which is 11 in base 6, then when you add 4 and 4 plus the one you get 9, which is 13 in base 6. You then add 3 and 1, plus the one that was carried over and get 5. The third problem is in base 5. You know this because, since you can’t do 0-3 you have to borrow, and to get 2, it has to be 5 that is borrowed, since 5-3=2.

29. The binary operation \( \heartsuit \) is defined by the set of shapes \{\( \bullet, \heartsuit, \spadesuit \)\} according to the table that follows. For example, \( \bullet \heartsuit \heartsuit = \spadesuit \). Is the operation closed? Is the operation commutative? Is the operation associative? Is there an identity shape?
• Concept- The concept of this problem is understanding the Cartesian product model of multiplication. You have to know how to do this to figure out how to multiply these and know what properties hold true to it.

• Answer- This operation is not closed. It is associative and commutative, and there is an identity shape, it is ✪. Associative means that grouping doesn’t matter. If you group them in different ways, you still get the same answer. For example, (✪ ♥ ◆) ♦ = ✪ and (◆♥♠) = ✪. The grouping changed, but the answer didn’t. Commutative means that order doesn’t matter. For example, ♦ ♥ ✪ = ♦ and ✪ ♥ ♦ = ♦. The identity shape is ✪ because anything multiplied by that is itself.

30. Rewrite each of the following in the form of a single exponential:
   i. 3²×3³ =?
   ii. 4²×5³ =?
   iii. (6²)⁶ =?
   iv. 10¹×10⁴ =?
   v. (12⁷)⁹ =?

• Concept- The concept of this is working with exponents. You have to now the rules of exponents to solve these.

• Answer- The answers to these can be solved by using the rules of exponents. If the bases are the same and the exponents are different, you add the exponents, if a power is raised to a power, you multiply the powers. If the exponents are the same and the bases are different, you multiply the bases. So the answers in order are: 3⁸, 20⁸, 6¹², 10⁵, and 12⁶³.
1. Suppose you have 4 mats, 22 strips, and 12 units; what is the smallest number of manipulative pieces one could have after making all exchanges?
   **ANSWER GUIDE:** Consider the mats as 100, strips as 10, and units as 1.

2. Joe works on 5 consecutive Tuesdays. On the first Tuesday Joe worked 3 hours 25 minutes, the second Tuesday he worked 7 hours 35 minutes, the third Tuesday he worked 2 hours 15 minutes, the fourth Tuesday he worked 8 hours and 38 minutes, and the last Tuesday Joe worked 10 hours and 5 minutes. How many 8-hour days, how many hours, and how many minutes total?
   **ANSWER GUIDE:** Add up the hours, and individually add the minutes and convert them to hours.

3. The following number is in base 3 notation. Use the number to then convert to base 6 notation.
   \[22102\]
   **ANSWER GUIDE:** Make sure you go to base ten first then to base 6 by dividing the number out.

4. Fill in blanks to find the sum of the following equation in base five notation. How do you know?
   \[
   4_2 \\
   + _34 \\
   \_
   4 1
   \]
   **ANSWER GUIDE:** Don't forget that \(4+1=0\), don't forget to carry the remainders.

5. What equivalence of fractions is shown in this pair of colored models?
   1st circle should have 4 shaded regions.
   ![Blue](Blue.png)
   **ANSWER GUIDE:** Use simplification

6. What properties would be useful to solve \(1/7 + 2/-4\) then solve.
   **ANSWER GUIDE:** Be sure to find a common denominator then solve regularly.

7. Suppose that \(\sqrt{7}\) is irrational. Explain why \(10+\sqrt{7}\) is also irrational.
   **ANSWER GUIDE:** Use the Suppose, then, and thus method.
8. Simplify the fraction and determine if they’re equivalent, how do you know?
   \( \frac{20}{44} = \frac{10}{22} \)
   **Answer Guide:** Simplify and look at the difference between the fractions, are they similar? How?

9. Consider the number \( \frac{4356}{260} \). Is this number rational or irrational? Why or why not?
   **Answer Guide:** Look at the prime factorization for the denominator and remember the 3 requirements for a number to be rational.

10. You win the lottery and you can take your money, but you have two different options in which to do so. One being the lump sum which is $10,000 which can be given to you at four different times during the year, but the interest rate would be 2%, or the lump sum of $10,000 given to you at only 2 different times but with an interest rate of 5%. Which is the best option to take and why?
    **Answer Guide:** Consider the equation \( P(1+r/100(1/n))^nt \)

11. Alex went to his grandmother’s farm. His grandmother has pigs and hens on her farm. He noticed that there were a total of 38 heads and 100 feet among them. How many hens and how many pigs did his grandmother have?
    **Answer Guide:** To solve this problem you can draw a model or create a table.

12. There are twenty-five students in a class. Each got an A, B, or a C for a test. Show that there are at least nine students who received the same grade.
    **Answer Guide:** To solve this problem you would use the Pigeonhole Principle.

13. In a race, the first five finishers in some order were a Honda, Jeep, Toyota, Dodge, and a Nissan.
    a. The Nissan finished 6 seconds before the Jeep.
    b. The Honda finished 5 seconds after the Dodge.
    c. The Toyota finished 7 seconds after the Dodge.
    d. The Jeep finished 3 seconds before the Honda.
    In what order did the cars finish the race?
    **Answer Guide:** To solve this problem you would set up a line with spaced points each representing 1-second time intervals.
14. Ryan, Alicia, and Tom play on the soccer team. Their positions are forward, midfielder, and goalie. Given the information below, determine who plays each position.
   - Ryan and the goalie bought ice cream for Alicia.
   - Ryan is not the goalie.

   **ANSWER GUIDE:** Solve this problem by creating a table of the possibilities.

15. Find the greatest common divisor of 270 and 360.
   **ANSWER GUIDE:** To solve this problem you can use one of two methods:
   - Prime factorization
   - Euclidean algorithm

16. Using the Euclidean algorithm, find the Least Common Multiple (LCM) of 2232 and 3828.
   **ANSWER GUIDE:** To solve this problem you would use the Euclidean algorithm.

17. Represent 500 as a product of prime factors.
   **ANSWER GUIDE:** To solve this, you can use one of two methods:
   - Create a factor tree
   - Use stacked short division

18. Test each number for divisibility by each of 2, 3, and 5.
   a. 1536
   b. 999
   c. 515
   d. 2310
   **ANSWER GUIDE:** To solve this series of problems you would look at the divisibility tests of 2, 3, and 5.

19. Michael worked 8 months out of the year. What percent of the year did he work? (Round answer to the nearest hundredth).
   **ANSWER GUIDE:** Set up an equation using proportions, then cross multiple. The final step would be to divide.

20. How would you use money to explain the decimal 47.65 to someone?
   **ANSWER GUIDE:** Write answer in expanded notation.
21. Conjecture how many cards are in this deck

Attribute cards can be made by drawing different shapes on cards carrying the figure shown, the color used, and so on.

**ANSWER GUIDE:** Different Varieties (Star+Moon+Sun) x3 at most possible shapes on one card x 6 different colors (Blue, Black, Red outlined or filled in) 3x3=9 9x6=54 cards

22. How many ways can four different pictures be arranged on a shelf?
**ANSWER GUIDE:** Make a diagram or a chart. Remember 4x3x2x1=24

23. Ashley has attempted a 3-digit subtraction problem. Identify what she is doing incorrectly.

905
-436
531

**ANSWER GUIDE:** She took the difference between the numbers

24. How many 7 hour days did George work?

- 5 Hours 25 Minutes
- 8 Hours 10 Minutes
- 6 Hours 45 Minutes
- 7 Hours 30 Minutes

**ANSWER GUIDE:** Remember add hours and minutes separately then convert minutes to hours. Worked 3-7 Hour shifts and had 6 Hours and 50 Minutes left

25. Finish labeling

Given:
Final Exam Homework Questions

4/15/13

n(\text{u})=100 \quad n(\text{A} \cap \text{B})=14
n(\text{A})=33 \quad n(\text{B} \cap \text{C})=9
n(\text{B})=51 \quad n(\text{A} \cap \text{C})=8
n(\text{C})=34 \quad n(\text{A} \cap \text{B} \cap \text{C})=3

\text{ANSWER GUIDE: } \text{Fill in the chart and subtract what is given to find what is needed}

26. If all the counters are used, list all the integers that can be represented with 10 counters.
   \text{ANSWER GUIDE:}
   \begin{itemize}
   \item Draw the circles
   \item Make a number line
   \item Make an orderly list
   \end{itemize}

27. If a < b, then a \leq b. True or false? TRUE
   \text{ANSWER GUIDE:}
   \begin{itemize}
   \item Make a number line
   \item Draw a picture
   \end{itemize}

28. Is \( \pi \) a rational or irrational number? Explain
   \text{ANSWER GUIDE: } \pi \text{ never repeats nor terminates so it is a irrational number}

29. Convert 5.4372 (372)Repeating
   \text{ANSWER GUIDE:}
   \begin{itemize}
   \item Set number = x
   \item Multiply both sides by 10 to line up the repeating decimal
   \item Multiply both sides by 1000 to get the whole repeating decimal on one side
   \item Subtract then divide by 9990 to get fraction
   \end{itemize}

30. If a:b=c:d then (a+b):b=(c+d):d Explain
   \text{ANSWER GUIDE: } \text{For example, } 3:5=6:10 \text{ already proportional so we know } a*d=b*c
1. Find A, B, and C by using guess and check.

-You are finding the answers to the little bubbles and you should be able to if you just use guess and check.

2. Find the answer to the sum of def+fed.

\[
\begin{align*}
\text{Abc} &= 624 \\
\text{Cba} &= -426 \\
\text{Def} &= 198
\end{align*}
\]

-Why do they equal the same number? Use inductive reasoning.

-The idea of inductive reasoning helps us to critically think why exactly this works the way it does.

3. Make a Venn Diagram and shade A U B U C (U=union).

-For this problem, to be able to fully grasp the concept, you have to understand key symbols and terms like union, intersecting, and compliment.

4. Find the all the possible sets of C×D if it equals 14.

\[
\begin{align*}
\text{C×D} &= 14 \\
\text{C} &= \{ \} \\
\text{The factors of 14 are 1, 2, 7, and 14.}
\end{align*}
\]
D= \{ \}
C= \{1 \ 2 \ 14\}
D= \{7\}

-This idea is dealing with finding all possible pairs of sizes of C and D and being able to put them into the sets.

5. What is $46_{10}+46_{10}$ if both are in base 5?

\[
\begin{array}{c c c}
46_{\text{base 5}} & + & 46_{\text{base 5}} \\
\hline
1 & + & 23 \\
& & 100
\end{array}
\]

- With this problem, you have to know it’s common adding, but that it’s in a different base and not its usual base 10.

6. If 30 is composite, is it true that all prime factors of 30 must not exceed $\sqrt{30}$?

- The whole idea of this problem is to find a counter example and see if the factors are greater than the $\sqrt{30}$. By finding a counter example, you will be able to find the answer.

7. Determine all the digits of $s$ so that $6,34s,217$ is divisible by 3.

- To figure out the answers to this problem, you have to be familiar with the divisibility chart, specifically for 3. The rule for 3 is the sum of the digits must be divisible by 3.

8. What can be added to the circle to get -7?

Black=positive
Red=negative

This is adding integers to get either the negative or positive number.

9. “3 by 6” piece of wood is planed from a board to the final size of 2½” by 5½”. Find the dimensions x and y of the shape of the two boards.
The concept of this problem is to use the formula $\frac{A}{b} = \frac{Ac+b}{c}$.

You also are trying to find $x$ and $y$. For $x$, you will use subtraction of fractions and for $y$, you will use addition. Then the formula to find the dimension.

10. What is a fraction by definition and what is the fraction and percent of this problem?

- The concept of this problem is to be able to identify the fraction represented by the shape and also be able to convert it to a percent once you found the fraction.

11. You and a friend are trying to decide who has to ride in the backseat on the trip to Florida so you decide that whoever draws the last straw out of 25 straws gets to ride in the front seat. When you draw straws you can draw between 1 and 4 straws each time. Is there a way to guarantee that either player 1 or player 2 can win every time? If so, is it player one or player two that can guarantee a win? How would they go about securing their win each time?

* Player 2 can win every time as long as they leave a multiple of five straws left after every turn. If they do this eventually there will only be 5 straws left
meaning that at most player one can take four still leaving one for player 2 to take.

12. Christopher reads 10 pages in his book every day, but when Christopher goes to bed at night he will forget the last 3 pages he read. So, the next day Christopher will begin reading his 10 pages with the 3 that he forgot while sleeping. If Christopher starts out on page 1 the first day on what day will he reach page 50?

*Christopher will reach page 50 on the 7th day because at the end of each night Christopher has only read +7 pages so on the 6th night Christopher will have read 42 pages and the next day, the 7th day, he will reach page 52.

13. Complete the following division operation in base 7.

$$23_7 \sqrt{2252_7}$$

*Please just pretend that the radical sign is a division bar.

The answer will be 66 remainder 16. To divide these numbers in base seven it will be helpful to first find all the multiples of 7 in base seven i.e. 23*0=0, 23*1=23, 23*2=46 23*3=101

14. A number is divisible by 3 if and only if the sum of all the digits in a number is divisible by 3. Explain why this works?

*Review the properties of divisibility (pages 203 and 204 in the book)

15. What is the gcd and lcm of 224 and 308?

Gcd (224, 308) = 16 Lcm (224, 308) = 2464

First find the gcd of 224 and 308 by using the Euclidean algorithm, then once you find the gcd to find the lcm you can use the equation \( \frac{a \cdot b}{\text{gcd}(a,b)} \).

16. If Sandy had to buy 12 bags of gravel to cover her driveway which is 50 ft by 20 ft then how many bags of gravel would she need to buy to cover a driveway that is 80 ft by 15 ft?

To solve this problem you will need to set up a proportion using the area of driveway #1 over 12 bags which is equal to the area of driveway #2 over x amount of bags and then cross multiply to solve.

17. Is \( 7 + \sqrt{5} \) irrational or rational? Explain why.

This will test your knowledge on rational and irrational numbers. The answer is that it will irrational because:

Suppose \( 7 + \sqrt{5} \) were rational then \( 7 + \sqrt{5} = \frac{a}{b} \). But then \( \sqrt{5} = \frac{a}{b} - 7 \) and \( \frac{a}{b} - 7 \) is rational. But the \( \sqrt{5} \) would be rational, a contradiction. Thus, \( 7 + \sqrt{5} \) is irrational.

18. Sarah had a coupon for 10% percent off any one item at a department store so she went shopping. Sarah found a shirt that was originally $50 but on this day was marked down to 20% off. How much will Sarah have to pay for the
$50 dollar shirt with its 20% mark down and the use of her 10% off coupon. (Hint the 10% off coupon will be used after the 20% markdown).
To complete this problem you will have to understand how to use percentages. First you will multiply $50 by .20 which will equal $10 meaning that you will get $10 off the $50 dollar shirt bringing your total to $40 then you will get an extra 10% off of that which will be $4 so the final cost of the shirt will be $36

19. A drawer of socks contains black and white socks. 1/3 of the socks are black and there are 10 more white socks than black socks. How many black socks and white socks are there in the drawer?
To solve this problem you will need to be familiar with working with fractions and using them to solve equations. Here the answer is 10 black socks for every 20 white socks.

20. Convert 3.14555….. to a fraction (the five is repeating, only the five. Sorry I couldn’t get a bar above the five, I tried literally everything.)
In order to do this you will need to review how you convert a repeating decimal to a fraction.
Hint: You will begin by multiplying 3.14555…. by 100
The answer should be \( \frac{2831}{900} \)
21. Let \( U = \{p, q, r, t, u, w, y\}\) be in the universe, and let \( A = \{s, q, w\}, B = \{q, r, u, w, u\} \) and \( C = \{p, t, w, u, y\} \). Locate all ten elements of \( U \) in a three-loop Venn Diagram, and then find the following sets.

a) \( A \cup C \)
b) \( A \cap B \)
c) \( A \cup C \) (line over \( C \))

to find these you would start with \( A \cup C \) which means you would look at everything that is in both \( A \) and \( B \) because \( U \) means union. For \( A \cap B \) you would look at the letters that are in BOTH \( A \) and \( B \). For \( A \cup C \) (line over \( C \)) everything that is in \( A \) but not in \( C \).

22. Convert \( 35710 \) to the corresponding base-four number.

first you would find the powers of 4 and then divide 35710 by 4 until you reach 0
to find what 35710 would be in base 4.

23. Show that if \( n^2 \) is even, then \( n \) is even.

24. Consider adding a 4 digit number by another 2 digit number base 7.
If you can use each of 0, 1, 2, 3, 4 and 5 exactly once, what is the smallest sum you can make?
Arrange the numbers in different combinations to see what number you can get that is the smallest.

25. Find the prime factorization of 4386 and determine how many divisors it has.
first you would use a factorization tree to factorize the number 4386 then you would multiply the power of the exponents to see how many different divisors 4386 would have.

26. Determine weather or not 27435 is divisible by 3 and 9.
To find this answer you would use the divisibility test of 3 and 9 which would be that $27435 = 2 \cdot 10000 + 7 \cdot 1000 + 4 \cdot 100 + 3 \cdot 10 + 5$ which equals $2 \cdot (9999 + 1) + 7 \cdot (999 + 1) + 4 \cdot (99 + 1) + 3 \cdot (9 + 1) = 5$. since 9999, 999, 99, and 9 are divisible by 9 and 9 is divisible by 3 then 27435 is divisible by 9 and 3.

27. Compute GCD (18, 411, 1649)
you would use the euclidean algorithm to find this answer.

28. Determine the absolute values of these integers
a) -90 b) 14 c) 0 d) -200
a) since -90 = absolute value of 90 = -(-90) = 90
b) since 14 is positive the absolute value of 14 is 14
c) the absolute value of 0 = 0
d) since -200 is negative, the absolute value of -200 = -(-200) = 200

29. What fraction is represented by the shaded areas in this picture?

first determine what fraction each picture is then add them. You may need to find a common denominator to add them.

10. How much would you have to invest at 6% interest compounded annually in order to have $18000 at the end of 3 years?
  to answer this question you would use the equation $18000 = p \cdot (1.06)^3$. 