MATHEMATICS FOR ELEMENTARY EDUCATION I (MATH 151)
SPRING 2015
EXAM I PROBLEM SET

Part I

(1) Perform the following addition problem via the standard algorithm. Explain how each step corresponds to a manipulation of manipulatives.

$$\begin{array}{cccc} & \mathrm{E} & ! & \& \\ + & \# & \# & \mathrm{E} \\ \hline \end{array}$$

(2) Perform the following subtraction problem via the standard algorithm. Explain how each step corresponds to a manipulation of manipulatives.

$$\begin{array}{cccc} & \mathrm{E} & @ & ! \\ - & ! & \# & \mathrm{E} \\ \hline \end{array}$$

(3) Perform the following multiplication problem via the standard algorithm. Explain how each step corresponds to a manipulation of manipulatives.

$$\begin{array}{cccc} & \mathrm{E} & @ & \& \\ \times & \# & ! & \mathrm{E} \\ \hline \end{array}$$

(4) Perform the following long division problem via the standard algorithm. Explain how each step corresponds to a manipulation of manipulatives.

$$\&!\,\overline{)\,\&\&!E}$$

(5) Two definitions of multiplication are $a \times b = \overbrace{a + a + \cdots a}^{b \text{ times}}$ and $a \times b = \overbrace{b + b + \cdots + b}^{a \text{ times}}$. Explain why these two are equivalent.

(6) Give a single picture that illustrates 3 groups of 5 and 5 groups of 3. Explain this picture's relationship to properties of multiplication.

(7) What does it mean to say that multiplication distributes over addition. State one of the definitions of multiplications and use this definition to explain why multiplication distributes over addition.

(8) The division $a \div b$ can be thought of as the number of groups formed by separating a group a objects into groups of b objects each or it may be thought of as the number of items in each group when separating a objects into b groups. Explain why this is and what this has to do with properties of multiplication.

(9) Draw a single picture that illustrates that

$$(\&\#) \times (!E) = (\&\times!) \times (!@)^2 + (\& \times E) \times (!@)^1 + (! \times \#) \times (!@)^1 + (\#\times!) \times (!@)^0.$$

(10) Explain using properties of multiplication and positional notation why

$$(\&\#) \times (!E) = (\&\times!) \times (!@)^2 + (\& \times E) \times (!@)^1 + (! \times \#) \times (!@)^1 + (\#\times!) \times (!@)^0.$$

(11) A pharmacist has a balance scale with 12 weights in total, 4 each of 1 gram, 5 grams, and 25 grams. The pharmacist measures out dosages by placing a collecting of their weights on one side of a balance scale and the dispensing the drug into the other side until the scale balances. What are all the weights the pharmacist can measure out?

(12) $a \times b = \&\&$, find all possible values of a and b.

(13) Perform the following addition problem using manipulatives. Explain all exchanges.

$$\begin{array}{cccc} & \mathrm{E} & ! & \& \\ + & \# & \# & \mathrm{E} \\ \hline \end{array}$$

(14) Perform the following subtraction problem using manipulative. Explain all exchanges.

$$\begin{array}{rccc} & \text{E} & @ & ! \\ - & ! & \# & \text{E} \\ \hline \end{array}$$

(15) Perform the following multiplication problem using manipulatives. Explain all exchanges.

$$\begin{array}{rccc} & \text{E} & @ & \& \\ \times & \# & ! & \text{E} \\ \hline \end{array}$$

(16) Perform the following long division problem using manipulatives. Explain all exchanges.

$$\&!\,\overline{\big)\&\&!E}$$

(17) Consider adding a three "digit" number to a two "digit" number in our one-handed arithmetic. If each of the symbols @, !, &, E, # is used only once, what is the biggest sum that can be achieved? How many ways can this sum be achieved? Be sure to explain your answer.

(18) Consider multiplying a three "digit" number to a two "digit" number in our one-handed arithmetic. If each of the symbols @, !, &, E, # is used only once, what is the biggest product that can be achieved? How many ways can this product be achieved? Be sure to explain your answer.

(19) Sally has 4 pennies, 4 nickels, and 4 quarters. What dollar amounts can she make?

Part II

(1) Consider the operation $\otimes$ defined by the table:

$\otimes$	$\diamondsuit$	$\clubsuit$	$\spadesuit$	$\heartsuit$	$\bigcirc$	$\square$
$\diamondsuit$	$\diamondsuit$	$\clubsuit$	$\spadesuit$	$\heartsuit$	$\bigcirc$	$\square$
$\clubsuit$	$\clubsuit$	$\spadesuit$	$\diamondsuit$	$\square$	$\heartsuit$	$\bigcirc$
$\spadesuit$	$\spadesuit$	$\diamondsuit$	$\clubsuit$	$\bigcirc$	$\square$	$\heartsuit$
$\heartsuit$	$\heartsuit$	$\bigcirc$	$\square$	$\diamondsuit$	$\clubsuit$	$\spadesuit$
$\bigcirc$	$\bigcirc$	$\square$	$\heartsuit$	$\spadesuit$	$\diamondsuit$	$\clubsuit$
$\square$	$\square$	$\heartsuit$	$\bigcirc$	$\clubsuit$	$\spadesuit$	$\diamondsuit$

For example $\spadesuit \otimes \heartsuit = \bigcirc$. What is the identity of this operation? Is this operation commutative? Give an example (not involving the identity) that shows this operation may be associative.

(2) Is subtraction associative over the integers? If so, why? If not give a counterexample.

(3) The size of $A \times B$ is 12, what are all the possible pairs of sizes of A and B? Choose one of these pairs, and appropriately sized sets A and B and give the elements of the set $A \times B$.

(4) Let the universe be students in this classroom. Let R be the set of students in the class that are right handed. Let B be the students in the class room where a blue shirt, and let K be the students in the classroom that are originally from Kentucky. Describe in words the set represented by $\overline{(B \cup \overline{K})} \cap R$ and draw a Venn diagram to represent that set.

(5) Let the universe be the set of whole numbers. Let F be the set of numbers with at most 3 factors, let O be the set of odd numbers, and let V be the set of numbers whose spelling ends in a vowel (a,e,i,o,u). Draw a Venn diagram for the set $\overline{(\overline{V} \cup O)} \cap F$ and describe the elements of that set in words. Hint: The set $\overline{(\overline{V} \cup O)} \cap F$ has only one element.

(6) onsider the binary operation on the integers defined by $a \oplus b = a + b - a \times b$. Note that $3 \oplus 4 = 3 + 4 - 12 = -5$. Is this operation closed? commutative? Does this operation have an identity, if so what? If this operation were defined only over the positive integers, which of your answers would change and how?

(7) Explain why $(-a) \times b = -(a \times b)$.

(8) Draw the general Venn diagram for three sets. Choose three sets A, B, and C so that in the universe of students in this classroom, there are likely to be students in each of the eight sets formed by A, B, and C.

(9) If a, b, and c are integers with $a < b$, when is it true that $ca > cb$?

(10) For any two integers a and b is it true that $|a + b| \leq |a| + |b|$? Why? When can the inequality be replaced with equality?

(11) We define an operation $\circ$ on the integers by $a \circ b = a + b + a \times b$. Are the integers closed under $\circ$? Is this operation commutative? Does this operation have an identity?

(12) If a and b are positive integers, explain why $a \times (-b) = -(a \times b)$.

(13) A five person team runs a 50 mile ultra-marathon in 5 hours, 23 minutes, and 28 seconds. Use the standard algorithm (long division) to find the average running time of the four runners. Explain what exchanges you did to solve this problem.