Part 1

Addition

Definition: Combining two or more groups of objects in a lump sum, so that the total is equal to the amount we started with.

Standard Algorithm:

- Align the numbers being added, one above the other, beginning with the right most place.
- Add the digits in the singletons place. When you have a number larger than #, you have to carry over to the next place. For example, !@ is one more than # and !@ singletons make ! stick. So, if you were to have !# singletons, you would leave the # in the singletons place, and carry the ! over to the sticks place.
- Move over to the sticks place. Add the sticks. Again, # is the largest number that can go in any one place. When you have an amount of sticks greater than #, you have to carry over to the flats place. For example, if you ended up with && sticks, you would keep & in the sticks place and carry & flats over to the flats.
- This process is repeated until you have the final sum.

Our one-handed notation is base 5. So the largest value that can go in any single place is 4. When you have one more than four (singletons), you can make one group of the next place value (sticks). So when you have more than four singletons, you have to carry the number of sticks you have over to the sticks place. This continues from sticks to flats and then to cubes and so on.

Important Properties: Addition is associative, meaning that 3+(5+6)=(3+5)+6=14. Moving parentheses or moving the numbers around does not affect the answer.

Subtraction

Definition: Taking an amount of objects away from a larger amount of objects. The inverse of addition.

Standard Algorithm:

- Align the values one above the other beginning with the right most place value, with the top number being the largest.
- Take the number of singletons in the bottom number away from the number of singletons in the top number.
- When there are not enough singletons in the top number to subtract from, you must borrow from the sticks place in order to make enough singletons to subtract from. When you borrow, you are borrowing ! stick and breaking that up into !@ singletons.
- Add those !@ singletons that you borrowed to the singletons already in the top number. Then, subtract the bottom number of singletons from the top number of singletons.
- Move to the sticks place. Subtract the number of sticks in the bottom number from the number of sticks in the top number. If you don't have enough sticks to subtract from, you must borrow from the flats. You are borrowing ! flat and breaking that up into !@ sticks.
- Add those !@ sticks to the number of sticks already in the top number, then subtract the number of sticks in the bottom number.

- Repeat this process for the flats, cubes, etc. until you have found the final difference.

This algorithm works for any positional notation system because when the number of objects you need to subtract is larger than the amount you have to subtract from, you can borrow from the next place value to make the amount large enough to subtract from. When you borrow, you are breaking up the higher-level object into the smaller unit and creating enough of the smaller unit to subtract from.

Important Properties: If a – b= c then b + c= a.

Multiplication

Definition: a x b is the area of a rectangle of size a length and b width

Standard Algorithm:

- Align the values one above the other beginning with the right most place. Typically, the larger of the numbers goes on top.
- Single digit multiplication can be thought of as repeated addition. For example, a x b = a b times, or a x b = b, a times.
- Multi-digit by single digit multiplication can also be thought of as repeated addition, however it can also be thought of as a quicker way to add large sums.
- Multiply the singletons first. Like addition, you have to carry anything larger than # over to the sticks place because # is the largest value you can have in any single place. For example, when multiplying & by E, you begin with & x E = !!. Since !! is greater than #, you leave ! in the singletons place and carry ! over to the sticks place.
- Next, you multiply # by E and add the sticks you've carried. So # x E = !& plus ! is !E. So your final answer for this example is !E!.

This works because, again, # is the largest value you can have in any single place. By carrying, you are taking the excess number of the smaller objects and grouping them to become one of the next largest items. Since !@ singletons makes ! stick, you carry ! over to the sticks place (according to the example above). This works for any positional notation system. No matter what the base is for the system you are using (base 5, base 7, base 10), you can take one of more than the maximum number of objects that can go in any one single place, and group those into one of the next largest item.

Important Properties: Multiplication is also associative, meaning that 2 x (3 x 4)=(2 x 3) x 4=24. It is also commutative, meaning that changing the order of the operation does not affect the answer (ex. a x b=b x a. Lastly, it is also distributive, meaning that 2 x (3+2)=(2 x 3) + (2 x 2).

Division

Definition: The number c so that b x c = a. If a is divided by b, then you are separating quantity a into b number of groups.

Standard Algorithm:

- A long division problem is set up using a bracket. The number being divided (dividend) goes under the bracket, and the number doing the dividing (divisor) goes on the outer left of the bracket. For example, 4)184.
- In the problem given as an example, you want to break the dividend into 4 equal groups (that's what the divisor tells you). To begin, you start with the hundreds place. 1 cannot be divided into 4 groups. So, theoretically, you would place a 0 above the bracket indicating that the hundreds cannot be broken down into 4 groups.
- You multiply the number you just placed above the bracket by the divisor, which tells you how many hundreds you've used, which is 0.
- Place the 0 below the 1 underneath the bracket and subtract. You are left with 1.
- Bring down the next number in the dividend. By doing this, you are breaking down the higher level number (the hundreds) and adding it to the next place value (the tens). Since there are 10 tens in a hundred, and you have broken that 1 hundred down into 10 tens, you add the 10 tens to the 8 tens you have brought down. This gives you 18 tens.
- Now you want to see if that 18 tens can be broken down into 4 groups (divisor). It can.
- 4 goes into 18 4 times. Place that 4 about the bracket in the tens place.
- Multiply that 4 by the divisor to see how many of the tens you've used, 16. Put the 16 tens below the 18 and subtract. You have 2 tens left over.
- Now, you break those 2 tens down into 20 ones by bring down the next number in the dividend, 4. Add those 20 ones to the 4 ones and you have 24 ones.
- 4 goes into 24, 6 times. Place that 6 above the bracket in the ones place and multiply it by the divisor to see how many ones you've used.
- Place that 24 below the 24 ones and subtract. You have nothing left over.

These steps work because when you are dividing, you are breaking up the dividend into smaller groups. You will have a number of groups that is equivalent to the divisor. The act of bringing down the next number in the dividend means that you are breaking down the higher level object into smaller pieces that are the same size as the objects in the next place value. This makes it easier to divide into groups. Sometimes, you may have a remainder of ones or singletons if that amount of ones or singletons cannot be broken down equally into the number of groups you are trying to create.

Important Properties: Something divided by 0 is undefined, and 0 divided by something is 0. For example, 2/0=undefined and 0/2=0. Also, anything divided by 1 is itself. For example, 3/1=3.

Part 2

• **If Jane has 16 marbles, Sally has 22 marbles, and Zack has 19 marbles, how many marbles would they have if they combined all of theirs together?**
For this problem, you would use the standard algorithm for addition. You are taking smaller groups of objects and turning them into one lump sum. 16 + 22 + 19 = 57

• **Lucy has 28 pieces of bubblegum. Her friends Joe and Nick each want 7 pieces. If Lucy gives each of them 7, how many pieces of bubblegum will she have left over?**
For this problem, you would use the standard algorithm for subtraction to solve. You want to break the total amount down into smaller groups. You would first subtract 7 from 28 to get 21. Then subtract 7 from 21 to get 14. So, Lucy would have 14 pieces left over.

• **William earns $650 a week at his job. If he works every week for a year, how much money will he make by the end of the year?**
For this problem you will use the standard algorithm for multiplication. Basically, you are shortening the time it would take to do repeated addition. To get the answer, you would multiply 650 by 52 because there are 52 weeks in the year. The answer would be 650 x 52=33,800.

• **Amber has 4 children. If she has $200 worth of allowance, and she wants to give each of her children an even amount of allowance, how much money will she give each child?**
For this problem you will use the standard algorithm for division. She has 4 children, so you want to break the $200 down into four equal amounts to determine how much child will receive. You will divide 200 by 4 to get 50. Therefore, each child will receive $50 in allowance.

• **Mackenzie walks into a store and picks up a bunch of apples. While she is shopping, she eats 2 of the apples. She checks out with 5 apples. How many apples did she pick up in the first place?**
This problem can be solved using addition. You are trying to determine how many apples she picked up to begin with, so you add the number of apples she ate to the number of apples she checked out with to get the total she began with. 2 + 5 = 7

Part 3

1. Train X departs from Grand Central Station at 11:35 am. Train Y departs from the same location at the same time. If Train X travels 137 miles north, and Train Y travels 452 miles south, how much distance will be between the two trains when they arrive at their destinations?

2. Solve { **! & # + & # @** }. Explain the carrying process.

3. Using manipulatives, solve **{ &!E / &** } and explain any exchanges you make.

4. a x b = b x a. Explain why this statement is true.

5. Multiplication can be defined as: a x b is the area of a rectangle that is a units wide and b units tall. Give an example that is representative of this definition and draw a picture to support your example.

6. Explain what it means to "borrow" when using the standard algorithm for subtraction. Give an example, or draw a picture using manipulatives as a visual aid.

7. Randi has 25 pieces of candy. He decides to share some with his friends. Afterwards, he is left with 16 pieces of candy. How much candy did Randi give away?

8. Solve **{ ## / & }** using manipulatives.

9. If a x b = #!, determine all the possible values of a and b.

10. Draw a picture that represents the problem **{ &@#! – E!# }.** Solve the problem using manipulatives and explain any exchanges you make.