Math 205 **Name:** ____________________

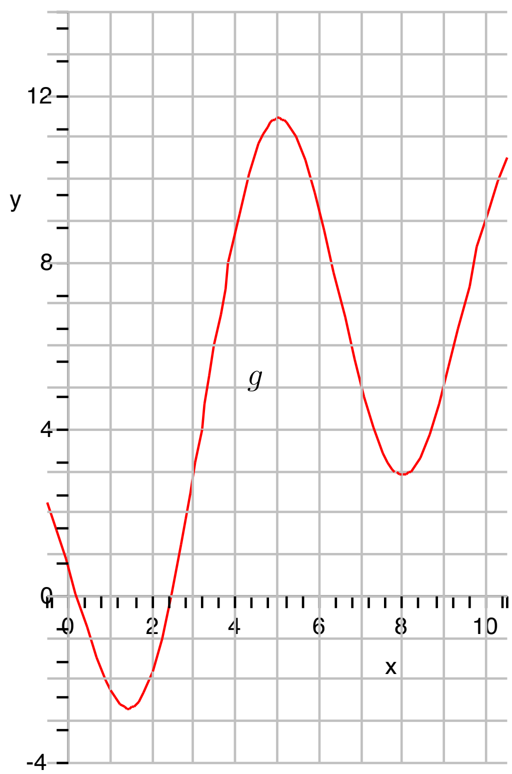
Question Bank 31 October 2012

1. Using the limit definition of the derivative show that $\frac{d}{dx}\sin(x) = \cos(x)$. It may be helpful to recall that $\sin(A + B) = \sin(A)\cos(B) + \cos(A)\sin(B)$.

2. Using differentiation rules find the first derivative of $\frac{\sin(x)(x^2+1)}{x^2+3x+1}$.

3. Alice and Bob are discussing the function $g(x) = \frac{x^2-3x-1}{(x-1)(x+3)}$. Bob claims that it has a root in $[-2, 4]$ because of the intermediate value theorem and the fact that $f(-2) = -3$ and $f(4) = \frac{1}{7}$. Alice agrees that there is a root in $[-2, 4]$ but says that Bob's reasoning is wrong. Why is Alice right?

4. Give give a plausible sketch of the derivative of g on the grid provided below. Be sure to explain you reasoning.



5. Let

$$f(x) = \begin{cases} x^2 - 4x + 3 & x < 4 \\ -x^2 + bx + c & x \geq 4 \end{cases}.$$

Choose b and c so that f is differentiable on $\mathbb{R}$.

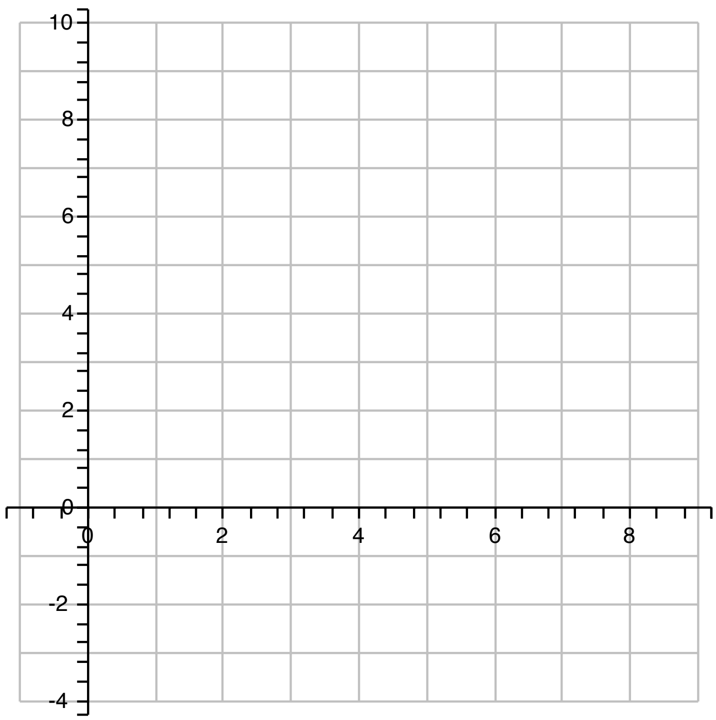
6. Find the derivative of $\cos(\sin(\tan(\theta)))$.

7. Evaluate (using limit laws) $\lim_{x\to 2} \frac{(x-2)(x+3)\cos(\pi x)}{(x-2)(x+4)} + \frac{\sqrt{x+2}}{x^2-4x+5}$.

8. Using two unknown coefficients, b and c, find all rational functions $f(x)$ with a horizontal asymptote of $y = 3$, vertical asymptotes of $x = 4$ and $x = -2$. Set up, but do not solve, equations that would allow you to solve for b and c in order to satisfy the additional conditions that $f(x)$ has a horizontal tangent at $x = 0$ and $x = -4$.

9. On the grid provided below sketch a function f defined on $[0, 8]$ so that

$$\lim_{x\to 3^-} f(x) = 2 \quad \lim_{x\to 3^+} f(x) = 4 \quad \lim_{x\to 5} f(x) = 6 \quad f(3) = 5 \quad f(5) = 3.$$



10. Find the secant lines for $f(x) = x^3 - 3x^2 + 1$ at $x = 2$ and the points $x = 3$ and $x = 1$. Give a reasoned guess as to the equation of the tangent line for $f(x)$ at $x = 2$.

11. Let

$$f(x) = \begin{cases} x^2 - 3x + 3 & x < 2 \\ -x^2 + bx + c & x \geq 2 \end{cases}.$$

Choose a and b so that f is continuous and $f(x) = 0$ has a root in $(2, 3)$.

12. Formally write what the statement $\lim_{h\to 0} \frac{(x+h)^2 - x^2}{h} = 2x$ means, then carefully show the statement is true using the limit laws. Use the formal defintion to explain why the choice of δ will not depend on the value of x?

13. Find a rational function f with vertical asymptotes $x = 4$ and $x = -2$, horizontal asymptote $y = 2$, and roots at $x = 3$ and $x = -1$.

14. $\lim_{x\to 2} \frac{x^3 - 2x - 4}{x - 2}$ represents the derivative of some function f at a point a. Find f and a and then find the tangent line to f at the point a.

15. Find the derivative of $f(x) = x^3 + 4x^2 + 3$ using the formal definition of a derivative.

16. Find the derivative of $f(x) = \frac{e^x(x^2+1)}{x^3-3x}$ by carefully applying the differentiation rules. Find where the derivative of f is equal to zero.

17. Find the first 3 derivatives of $\sec(x)$.

18. If we know that

$$\begin{array}{lllllll} f(-3)=3 & f(-2)=1 & f(-1)=0 & f(0)=-2 & f(1)=1 & f(2)=3 & f(3)=2 \\ g(-3)=-3 & g(-2)=-3 & g(-1)=2 & g(0)=-1 & g(1)=0 & g(2)=2 & g(3)=0 \\ h(-3)=1 & h(-2)=2 & h(-1)=3 & h(0)=-3 & h(1)=3 & h(2)=-2 & h(3)=1 \\ f'(-3)=0 & f'(-2)=0 & f'(-1)=1 & f'(0)=2 & f'(1)=-3 & f'(2)=3 & f'(3)=2 \\ g'(-3)=-3 & g'(-2)=-1 & g'(-1)=1 & g'(0)=1 & g'(1)=3 & g'(2)=2 & g'(3)=2 \\ h'(-3)=3 & h'(-2)=0 & h'(-1)=1 & h'(0)=-3 & h'(1)=-3 & h'(2)=1 & h'(3)=3 \end{array}$$

evaluate $\frac{d}{dx} f(g(h(x)))$ at $x = -1$.

19. Use the definition of $\tanh(x)$ to show that the derivative of $\sqrt[4]{\frac{1+\tanh(x)}{1-\tanh(x)}} = \frac{1}{2}e^{\frac{x}{2}}$.

20. A four sided figure is formed by the lines tangent to the points $(3, 4)$, $(-3, 4)$, $(-3, -4)$, and $(3, -4)$ on a circle of radius five centered at the origin. What is the area of the figure?

21. Find the derivative of $\frac{x^{\sin(x)}\sqrt{x^2+4}}{\ln(x)}$ without using the quotient or product rules.

22. The half-life of radium is 1590 years. Assuming you start with 200 mg of radium, how long will it be until you have 199 mg of radium?

23. Find the line tangent to the curve $y^2 = x^3 - 3x - 2$ at $(3, -4)$.

24. If the derivative $\frac{dx}{dy}$ exists and is zero at a point, then the tangent line is vertical. Find all six points on the curve $y^4 + 1 = y^2 + x^2$ where the tangent line is vertical.

25. Identify where the function $f(x) = 8 + 36x + 3x^2 - 2x^3$ is increasing, decreasing, concave up, and concave down. In addition find the local maximum and minimum values.

26. Show that the equation $x^5 + e^x = 0$ has exactly one real root.

27. The implicitly defined curve $y^4 + x^2 + xy = 13$ goes through the point $(3, 1)$. Use the linearization of the curve to estimate the y coordinate of the curve when $x = 3.1$

28. The volume of a cone is $\frac{1}{3}bh$ where b is the area of the base. Suppose that the radius and height of a circular cone are increasing at a rate of $2\frac{cm}{s}$. How fast is the volume of the cone increasing when the radius is $20cm$ and the height is $10cm$?

29. If f is a linear function, which values c on the interval $[a, b]$ satisfy the conclusions of the Mean Value Theorem.

30. Find the extreme values of $4t - \sqrt{t^2 + 1}$ on the interval $[-4, 4]$.

31. Show that for any real number k, $(1 + x)^k$ is approximately $1 + kx$ for small x.

32. As a man walks away from a 15 foot lamppost the tip of his shadow moves three times as fast as he does. What is the man's height?

33. Find the horizontal tangents of $3x^2 + 4y^2 + 3xy = 24$.

34. Find $\lim_{x\to\infty} \frac{(\ln(x))^p}{x}$ where p is any positive number.

35. Find $\lim_{x\to 0} \tan(3x)\csc(4x)$.

36. Sketch a curve with slant asymptote $3x - 2$ as $x \to \infty$, horizontal asymptote $y = 2$ as $x \to -\infty$, vertical asymptote $x = 2$, which is concave up on $(-\infty, -2) \cup (2, \infty)$, concave down on $(-2, 2)$, increasing on $(-\infty, -1) \cup (4, \infty)$ and decreasing on $(-1, 2) \cup (2, 4)$.

37. According to postal regulations a carton can be shipped through the mail only if the sum of its height and girth (the perimeter of its base) does not exceed 108 in. Find the dimensions of the carton of maximum volume that can be shipped if the base is square.

38. Use two steps of Newton's method to estimate a root of $x^2 - 6x + 8$ starting at $x = 1$.

39. Find the position of a particle at time $t = 2$ if its acceleration is given by $s(t) = t^3 - 5t + 6$ and it starts at rest at the origin.

40. Write down the infinite Riemann for the area under the curve $y = x^2$ from $x = -2$ to $x = 2$. Approximate this Riemann sum using 4 intervals.

41. Suppose we know that $\int_3^5 f(x)dx = 4$ and $\int_1^5 g(x)dx = -2$, and $\int_3^1 f(x)dx = 2$. Use this information to find $\int_1^5 4f(x) - 7g(x)dx$.

42. Evaluate $\int_1^2 \frac{x+3}{(x^2+6x+1)^3}dx$

43. Identify where $\int_0^x t^2 - 5t - 6dt$ is concave up, concave down, increasing and decreasing.