HW#2:

12.5 12, 22, 24, 26, 30, 48, 54, 60
11.1 12, 14, 26, 38
13.1 6, 7, 9, 18, 26, 32
13.2 10, 12, 20, 22, 26, 28, 32, 40, 43, 50, 56

$\boxed{12.5}$

12) $P = (1,0,0)$
$Q = (0,1,1)$
$R = (2,0,1)$

$\overrightarrow{PQ} = \langle -1, 1, 1\rangle$
$\overrightarrow{PR} = \langle 1, 0, 1\rangle$

$$\begin{vmatrix} i & j & k \\ -1 & 1 & 1 \\ 1 & 0 & 1 \end{vmatrix} = i(1) - j(-2) + k(-1)$$
$$= \boxed{\langle 1, 2, -1\rangle = n}$$

$n \cdot \overrightarrow{OP} = d$
$\langle 1,2,-1\rangle \cdot \langle 1,0,0\rangle = \boxed{1 = d}$
$n \cdot \langle x, y, z\rangle = d$

22) $n = \langle 1,1,1\rangle$
$n \cdot \overrightarrow{OP} = d$
$d = \langle 1,1,1\rangle \cdot \langle 4,1,9\rangle = 14$
$\langle 1,1,1\rangle \langle x,y,z\rangle = 14$

24) $n = \langle 1,0,1\rangle \quad P = (-2,-3,5)$
$d = \langle 1,0,1\rangle \cdot \langle -2,-3,5\rangle$
$= -2 + 5 = 3$
$\vec{n} \cdot \overrightarrow{OP} = 3$

26) $P = (-1, 0, 1)$
$r(t) = \langle t+1, 2t, 3t-1\rangle$
$= \langle 1, 0, -1\rangle + t\langle 1, 2, 3\rangle$
$Q = (1, 0, -1)$
~~$\overrightarrow{PQ} = (2, 0, -2)$~~
$\overrightarrow{QP} = (-2, 0, 2)$

$$\overrightarrow{QP} \times \vec{v} = \begin{vmatrix} i & j & k \\ -2 & 0 & 2 \\ 1 & 2 & 3 \end{vmatrix} = -4i - j(-6-2) + k(-4)$$
$$= \langle -4, 8, -4\rangle = \vec{n}$$

$\langle -4,8,-4\rangle \cdot \langle -1,0,1\rangle = 4 - 4 = 0$
$\langle -4,8,-4\rangle \cdot \langle x,y,z\rangle = 0$

30)
$x + y + z = 14. \quad r(t) = \langle 1,1,0\rangle + t\langle 0,2,4\rangle$
$r(t) = \langle 1, 1+2t, 4t\rangle$
$1 + 1 + 2t + 4t = 14$
$6t = 12$
$t = 2$
$r(2) = \langle 1, 5, 8\rangle$

~~5~~ 4

$$\overset{P}{(1,0,0)}, \overset{Q}{(0,1,0)}, \overset{R}{(0,0,1)}$$

yz-plane

$\overrightarrow{PQ} = (-1, 1, 0)$

$\overrightarrow{PR} = (-1, 0, 1)$

$$\begin{vmatrix} i & j & k \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{vmatrix} = i(1) - j(-1) + k(1)$$

$$= \langle 1, 1, 1 \rangle = n_1$$

yz-plane $\Rightarrow n_2 = \langle 1, 0, 0 \rangle$

$n_1 \cdot n_2 = \|\langle 1,1,1\rangle\| \|\langle 1,0,0\rangle\| \cos\theta$

$1 = (\sqrt{3})(\sqrt{1}) \cos\theta$

$\frac{1}{\sqrt{3}} = \cos\theta$

$\cos^{-1}\left(\frac{1}{\sqrt{3}}\right) = \theta$

11.1

12

60

$2x + y - 3z = 0$

$x + y = 1$

$y = 1 - x$

$2x + 1 - x - 3z = 0$

$3z = x + 1$

$z = \frac{x+1}{3}$

$$\boxed{\begin{aligned} x(t) &= t \\ y(t) &= 1 - t \\ z(t) &= \frac{t+1}{3} \end{aligned}}$$

11.1

12) $x = 1 + t^{-1}, \ y = t^2$

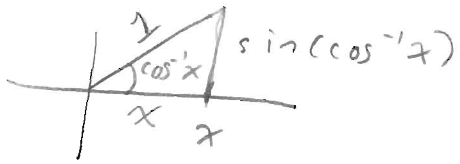
$x - 1 = \frac{1}{t}$

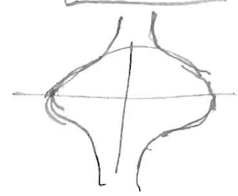
$t = \frac{1}{x-1}$

$y = (x-1)^{-2}$

14) $x = \cos t \quad y = \tan t$

$t = \cos^{-1} x \qquad y = \frac{\sin(\cos^{-1} x)}{\cos x} = \boxed{\frac{\sqrt{1-x^2}}{x}}$





$1^2 = x^2 + y^2$

$1^2 = x^2 + (\sin(\cos^{-1} x))^2$

$\sqrt{1-x^2} = \sin(\cos^{-1} x)$

$x^2 y^2 = 1 - x^2$

28) $x^2 + y^2 = 49$

$x = 7 \cos t$

$y = 7 \sin t$

$x = \pm\sqrt{49 - y^2}$

38) $y = x^2 \qquad c(0) = (3, 9)$

$x(t) = t + 3$

$y(t) = (t+3)^2$

13.1

6

7

9

18

$y^2 - z^2 = x - 2$

$y^2 + z^2 = 9$

$z = \sqrt{9 - y^2}$

$x = 2 + y^2 - (9 - y^2)$

$= -7 + 2y^2$

$\left(-7 + 2t^2, t, \pm\sqrt{9 - t^2}\right)$ $\quad -3 \le t \le 3$

26

$r(t) = t\langle 1, 0, 4\rangle + (1-t)\langle 4, 1, 2\rangle$

$= \langle 4, 1, 2\rangle + t\langle -3, -1, 2\rangle$

$= \langle 4 - 3t, 1 - t, 2 - 2t\rangle$ $\quad (t \mapsto -t + 1)$

32

$z = x^2 - y^2$

$z = x^2 + xy - 1$

$\cancel{x^2 - y^2 = x^2 + xy - 1}$

$x = t \quad 0 = y^2 + xy - 1$

$\cancel{t = x + y}$

$0 = y(y + x) - 1$

$\frac{1}{y} = y + x \quad (y \neq 0!)$

$x = \frac{1}{y} - y$

$\cancel{(x+y)(x-y) = z}$

$\cancel{x(x+y) - 1 = z}$

$y = t$

$x = \frac{1}{t} - t$

$z = \left(\frac{1}{t} - t\right)^2 - t^2$

$= \frac{1}{t^2} - 2 + \cancel{t^2} - \cancel{t^2}$

$= \frac{1}{t^2} - 2$

13.2

10

$r(\theta) = \langle \tan\theta, 4\theta - 2, \sin\theta\rangle$

$r'(\theta) = \langle \sec^2\theta, 4, \cos\theta\rangle$

12

$c(t) = t^{-1} i - e^{2t} k$

$= \left\langle \frac{1}{t}, 0, -e^{2t}\right\rangle$

$c'(t) = \left\langle -\frac{1}{t^2}, 0, -2e^{2t}\right\rangle$

20

$\frac{d}{dt}\left(t^4 r_1(t)\right)$

$4t^3 \cdot r_1(t) + t^4 r_1'(t)$

$4t^3\langle 8t, 4, -t^3\rangle + t^4\langle 8, 0, -3t^2$

22

$\boxed{13.2}$

22) $\frac{d}{dt}\left(r_1(t)\cdot r_3(t)\right)\Big|_{t=5}$

$= \left[r_1'(t)\cdot r_3(t) + r_1(t)\cdot r_3'(t)\right]_{t=5}$

$= r_1'(5)\cdot\langle 3,1,2\rangle + r_1(5)\cdot\langle -1,2,7\rangle$

$\vdots$

26) ~~r(t) =~~

$\frac{d}{dt}\vec{F}(g(t)) = g'(t)\,\vec{r}\,'(g(t))$

...

28)

32) $r(t) = \langle t^2, t^3, e^t\rangle$

$\frac{d}{dt}(r\times r') = r(t)\times r''(t)$

...

$=$

40) $\int_0^1 \langle 2t, 4t, -\cos 3t\rangle\, dt$

45)

50) $\frac{dr}{dt} = \langle e^{2t}, e^t, e^{-2t}\rangle$

$r = \left\langle \frac{e^{2t}}{2} + A,\ e^t + B,\ \frac{e^{-2t}}{-2} + C\right\rangle$

$r(0) = \langle \frac{1}{2}+A,\ 1+B,\ -\frac{1}{2}+C\rangle = \langle 4,-2,3\rangle$ ✓

$A = 3.5 \quad B = -3 \quad C = 3.5$

56 $L(t) = r(t_0) + t r'(t_0)$

$r(t_0) = \langle 5 - t_0, 21 - t_0^2, 3 - \frac{t_0^3}{27} \rangle$

$r'(t_0) = \langle -1, -2t_0, -\frac{t_0^2}{9} \rangle$

$0 = r + t r' = \langle 5 - t_0 - t, 21 - t_0^2 - 2t_0 t, 3 - \frac{t_0^3}{27} - \frac{t_0^2}{9} t \rangle$

$\cancel{= \langle 5 - 2t, 21}$

$5 - t_0 - t = 0$

$21 - t_0^2 - 2t_0 t = 0$

$3 - \frac{t_0^3}{27} - \frac{t_0^2}{9} t = 0$

$\cancel{t_0 t = t_0 - 5}$ $\boxed{t = 5 - t_0}$

$21 - t_0^2 - 2t_0(t_0 - 5) = 0$

$21 - t_0^2 - 2t_0^2 + 10t_0 = 0$

$-3t_0^2 + 10t_0 + 21 = 0$

$t_0 = \frac{-10 \pm \sqrt{100 - 4(-3)(21)}}{-6}$

$=$

Ach!

$63 \times 4 = 252$

252: 2, 126
126: 2, 63
63: 3, 21
21: 7, 3

5b) cont...

$$21 - t_0^2 - 2t_0(5 - t_0) = 0$$

$$21 - t_0^2 - 10t_0 + 2t_0^2 = 0$$

$$t_0^2 - 10t_0 + 21 = 0$$

$$(t_0 - 7)(t_0 - 3) = 0$$

If $t_0 = 3$ $\Rightarrow$ $t = 5 - 3 = 2$

(3): $3 - 1 - 1(2) = 0$ ✓

If $t_0 = 7$ $\Rightarrow$ $\cancel{t}$ $t = 5 - t_0$

$$= 5 - 7$$

$$= -2$$

$\Rightarrow$ at time $t_0 = 7$, the plane would have to fire behind itself to hit the origin. Not possible! So $t_0 = 7$ doesn't work.