~~4.3~~ ~~5.3~~

1

topographical map - google

composition of functions?

HW4

14.2	14.3		
~~4~~	~~4~~	~~48~~	~~68~~
~~8~~	~~8~~	~~56~~	~~74~~
~~14~~	~~40~~	~~58~~	76 c) d)
~~16~~	~~42~~	~~59~~	
~~18~~	~~44~~	~~64~~	
~~26~~	~~46~~	~~66~~	

$\boxed{14.2}$

4) $\lim_{(x,y)\to(-2,4)} \overbrace{\dfrac{x^2-3y^2}{4x+y}}^{f(x,y)} = L$ By Quotient law & def of continuity:

$f(x,y)$ is discontinuous when $4x+y=0$

$\iff y=-4x$

The point $(-2,4)$ is not on this line.

So you can just plug in:

$$L=\frac{(-2)^2-3(4)^2}{4(-2)+4}=\frac{4-3\cdot4^2}{-8+4}=\frac{\cancel{4}-3\cdot4^{\cancel{2}}}{\cancel{-4}}=\frac{1-3\cdot4}{-1}$$

$=11$

8) $\lim_{(x,y)\to(0,0)} e^{x^2-y^2} = L$

$f(z)=e^z$ is continuous everywhere

$g(x,y)=x^2-y^2$ is continuous

So $f(g(x,y))$ is continuous everywhere.

So just plug in:

$L=e^{0-0}=1$

5

$\boxed{14.2}$

14) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2+y^2}{1+y^2}$

Let $f(x,y) = \frac{x^2+y^2}{1+y^2}$

By the quotient ~~rule~~ law, f is continuous when $1+y^2 \neq 0$. Plug $(x,y)=(0,0)$ into this equation and see:

$$1+0^2 = 1 \neq 0.$$

So $f(x,y)$ is continuous at $(0,0)$.

Plugging in: $\frac{0^2+0^2}{1+0^2} = \boxed{0}$

16) $\displaystyle\lim_{(x,y)\to(-1,-2)} \frac{xy^2}{|x|} = L$

Let $f(x,y) = \frac{xy^2}{|x|} = \frac{x}{|x|} \cdot y^2$

~~Let g~~

Now $\frac{x}{|x|}$ is continuous when $x \neq 0$.

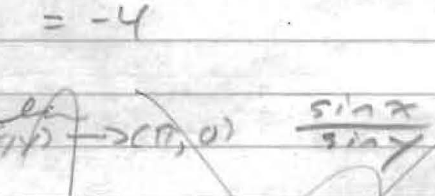
And y^2 is " everywhere.

As in example 3:

$$L = \left(\lim_{x\to -1} \frac{x}{|x|}\right)\left(\lim_{y\to -2} y^2\right)$$
$$= \frac{-1}{|-1|}\cdot(-2)^2$$
$$= \frac{-1}{1}\cdot 4$$
$$= -4$$

18) $\displaystyle\lim_{(x,y)\to(\pi,0)} \frac{\sin x}{\sin y}$ For $0 \le x \le \pi$

$\sin x = \sin(\pi - x)$



So for $0 \le x, y \le \pi$ $f(x) := \frac{\sin x}{\sin y} = \frac{\sin(\pi - x)}{\sin y}$

So $\displaystyle\lim_{(x,y)\to(\pi,0)} \frac{\sin x}{\sin y} = \lim_{(x,y)\to(\pi,0)} \frac{\sin(\pi - x)}{\sin y}$ on $0 \le x, y \le \pi$

$\boxed{14.2}$

18 $\lim_{(x,y)\to(\pi,0)} \frac{\sin x}{\sin y}$

The limit does not exist because $\frac{\sin x}{\sin y}$ is undefined on the x-axis. ($\leftarrow y=0$ axis)

(So every punctured disc around $(\pi, 0)$ contains a point where $\frac{\sin x}{\sin y}$ is undefined)

26 $\lim_{(x,y)\to(0,0)} \sin x \cos \frac{1}{y}$

$= \left(\lim_{x\to 0} \sin x\right)\left(\lim_{y\to 0} \cos \frac{1}{y}\right)$ if both limits exist

But $\lim_{y\to 0} \cos\frac{1}{y}$ does not exist.

~~Consider the sequence~~

So let's try something different:

$\sin t$ and $\cos t$ are bounded!

In fact, $-1 \le \sin t \le 1$ and $-1 \le \cos t \le 1$

So $|\sin t| \le 1$ and $|\cos t| \le 1$

$\cancel{\sin x \cos \frac{1}{y} \le \sin x \cdot 1 = \sin x}$

~~and~~ $\cancel{\sin x \cos \frac{1}{y} \ge \sin x \cdot (-1) = -\sin x}$

~~So~~ $\cancel{-\sin x \le \sin x \cos \frac{1}{y} \le \sin x}$

So $|\sin x \cos \frac{1}{y}| \le |\sin x||\cos x| \le |\sin x| \cdot 1 = |\sin x|$

So

$\lim_{(x,y)\to(0,0)} |\sin x \cos \frac{1}{y}| \le \lim_{x\to 0} |\sin x| = 0$

So $L = 0$.

14.3

4) $\frac{\partial}{\partial u} \ln(u^2+uv) = \frac{1}{u^2+uv} \cdot (2u+v)$

8) $\frac{\partial f}{\partial x}$ positive $\frac{\partial f}{\partial y}$ negative

40) $f_x(1,4)$

$f(x,y) = 3x^2y + 4x^3y^2 - 7xy^5$

$f_x(x,y) = 6xy + 12x^2y^2 - 7y^5$

$f_x(1,4) = 6(1)(4) + 12(1)^2(4)^2 - 7(4)^5 \ldots$

42) $g(u,v) = u\ln(u+v) \quad g_u(1,2)$

$g_u(u,v) = u\left(\frac{1}{u+v}\right) + \ln(u+v)$

$g_u(1,2) = 1\left(\frac{1}{1+2}\right) + \ln(1+2) \ldots$

44) $h(x,z) = e^{xz - x^2z^3}, \quad h_z(1,0)$

$h_z(x,z) = (x - 3x^2z^2)\exp(xz - x^2z^3)$

$h_z(1,0) = \ldots$

46) $P(V,T) = \frac{nRT}{V}$

$\frac{dP}{\partial T}(V,T) = \frac{nR}{V} \qquad \frac{\partial P}{\partial V}(V,T) = nRT(-V^{-2})$

48) An increase in r is my guess, and ...

$\frac{\partial V}{\partial r} = \frac{2\pi}{3} rh \qquad \frac{\partial V}{\partial h} = \frac{\pi}{3} r^2$

So $\frac{\partial V}{\partial r}(12,12) = \frac{2\pi}{3}(12)^2 \quad \frac{\partial V}{\partial h} = \frac{\pi}{3}(12)^2$

So the increase is twice as fast in the r direction.

$\boxed{14.3}$

56) $g(x,y) = xy\,e^{-y}$, $g_{yy}(1,0)$

$g_y(x,y) = x e^{-y} + xy\, e^{-y}(-1)$
$= x(1-y)e^{-y}$

$g_{yy}(x,y) = x(1-y)e^{-y}(-1) + x(-1)e^{-y}$

$g_{yy}(1,0) = \cdots$

58) $h(x,y) = \ln(x^3+y^3)$, $h_{xy}(x,y)$

$h_x(x,y) = \dfrac{1}{x^3+y^3}(3x^2)$

$h_{xy}(x,y) = (3x^2)\dfrac{-1}{(x^3+y^3)^2}(3y^2)$

59)

64) $g(x,y,z) = x^4y^5z^6$
$g_x = 4x^3y^5z^6$
$g_{xx} = 12x^2y^5z^6$
$g_{xxy} = 60x^2y^4z^6$
$g_{xxyz} = 360\,x^2y^4z^5$

66) $\cdots$

68) $R(u,v,w) = \dfrac{u}{v+w}$, R_{uvw}

$R_u = \dfrac{1}{v+w}$
$R_{uv} = \dfrac{-1}{(v+w)^2}$
$R_{uvw} = \dfrac{1}{(v+w)^3}$

$\boxed{14.3}$

74) $u(x,t) = \sin(nx)e^{-n^2t}$ Show $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$.

$\frac{\partial u}{\partial t} = \sin(nx)e^{-n^2t}(-n^2)$

$\frac{\partial u}{\partial x} = n\cos(nx)e^{-n^2t}$

$\frac{\partial^2 u}{\partial x^2} = -n^2\sin(nx)e^{-n^2t}$

76) a) $u(x,y) = x$

$u_x = 1$ $u_y = 0$

$u_{xx} = 0$ $u_{yy} = 0$

$\Delta u = 0 + 0 = 0$

b) $u(x,y) = e^x\cos y$

$u_x = e^x\cos y$ $u_y = e^x(-\sin y)$

$u_{xx} = e^x\cos y$ $u_{yy} = -e^x\cos y$

$\Delta = 0$

c) $u(x,y) = \tan^{-1}\frac{y}{x}$ $\frac{\partial}{\partial x}(\tan^{-1}x) = \frac{1}{1+x^2}$

$u_x = \frac{1}{1+(\frac{y}{x})^2}\cdot\left(-\frac{y}{x^2}\right)$

$\cancel{u_{xx}} = \frac{-y}{x^2+y^2}$ ...

d) $u(x,y) = \ln(x^2+y^2)$

$u_x = \frac{1}{x^2+y^2}(2x)$

$u_{xx} = \frac{1}{(x^2+y^2)^2}(2x)^2 + \frac{1}{x^2+y^2}(2)$

...

14.3

59 $\frac{\partial \rho}{\partial T} = 27.34 - 27.18 = .16$

$\frac{\partial \rho}{\partial T} = 28.01 - 27.18 = - - -$

etc - - -