We want to integrate the function f over the region bounded by $z = 4 - y^2$, $y = 2x$, $z = 0$, and $x = 0$ (For reference, below these will be the red, blue, green and magenta constraints.) In order to figure out what the shape looks like and how to parameterize the shape, we consider the picture with certain variables fixed and parameterize the region.

y FIXED

From Figure 1 we see that the region with y fixed is a rectangle. If $y \geq 0$, then this rectangle is bounded in the x direction by $x = 0$ and $x = \frac{y}{2}$ and in the z direction by $z = 0$ and $z = 4 - y^2$. Note that for x these bounds correspond when $y = 0$ and for z they correspond when $y = \pm 2$. Thus there are 2 possible regions, one where

$$\begin{aligned} -2 \leq y &\leq 0, \\ 0 \leq z &\leq 4 - y^2, \\ \frac{y}{2} \leq x &\leq 0, \end{aligned}$$

and another region where

$$\begin{aligned} 0 \leq y &\leq 2, \\ 0 \leq z &\leq 4 - y^2, \\ 0 \leq x &\leq \frac{y}{2}. \end{aligned}$$

For the first region the iterated integral is

$$\int_{-2}^{0} \int_{0}^{4-y^2} \int_{\frac{y}{2}}^{0} f(x, y, z) dxdzdy = \int_{-2}^{0} \int_{\frac{y}{2}}^{0} \int_{0}^{4-y^2} f(x, y, z) dzdxdy.$$

For the second region the iterated integral is

$$\int_{0}^{-2} \int_{0}^{4-y^2} \int_{0}^{\frac{y}{2}} f(x, y, z) dxdzdy = \int_{0}^{2} \int_{0}^{\frac{y}{2}} \int_{0}^{4-y^2} f(x, y, z) dzdxdy.$$

z FIXED

Now consider the case where z is fixed. From Figure we see again that there are two possible regions (as we should expect from the previous case, since the shape is the same in all instances). In this case y is bounded by $y^2 = 4 - z$ and $y = 2x$ and x is bounded by $y = 2x$ and $x = 0$. Note that this region becomes trivial only when $(x, y) = (0, 0)$ and hence $z = 4$. Thus

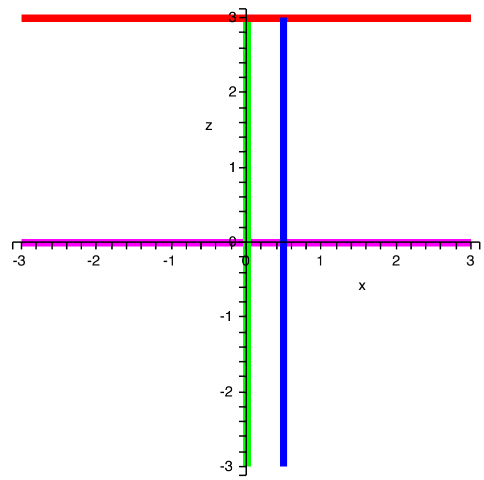

FIGURE 1. Bounding functions with $y = 1$

we have the following description of the regions,

$$
\begin{aligned}
&0 \le z \le 4, \\
&0 \le y \le \sqrt{4 - z}, \\
&0 \le x \le \frac{y}{2},
\end{aligned}
$$

and

$$
\begin{aligned}
0 &\le z \le 4, \\
-\sqrt{4 - z} &\le y \le 0, \\
\frac{y}{2} &\le x \le 0.
\end{aligned}
$$

For the first region the integral is

$$\int_0^4 \int_0^{\sqrt{4-z}} \int_0^{\frac{y}{2}} f(x, y, z) dx dy dz = \int_0^4 \int_0^{\frac{\sqrt{z-4}}{2}} \int_{2x}^{\sqrt{4-z}} f(x, y, z) dy dx dz.$$

For the second region the integral is

$$\int_0^4 \int_{-\sqrt{4-z}}^0 \int_{\frac{y}{2}}^0 f(x, y, z) dx dy dz = \int_0^4 \int_{\frac{-\sqrt{4-z}}{2}}^0 \int_{-\sqrt{4-z}}^{2x} f(x, y, z) dy dx dz.$$

x FIXED

Finally consider the case of a fixed x, again we see there are two possible regions. In both cases the z term is bounded by $z = 0$ and $z = 4 - y^2$ and the y term is bounded by $y = 2x$ and $z = 4 - y^2$. These regions only

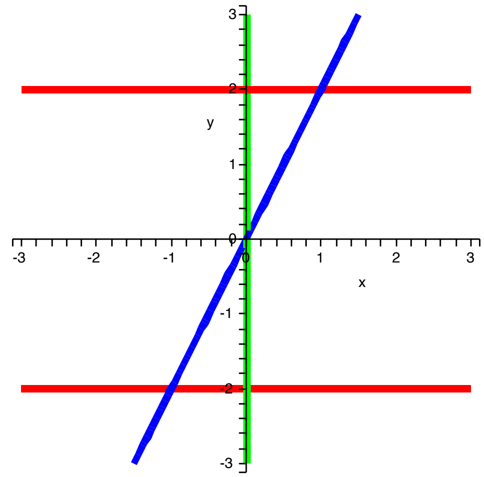

FIGURE 2. Bounding functions with $z = 0$

become trivial when $x = \pm 1$, and so again we have the descriptions of the two regions

$$\begin{aligned} 0 &\le x \le 1, \\ 0 &\le y \le 2x, \\ 0 &\le z \le 4 - y^2, \end{aligned}$$

and

$$\begin{aligned} -1 &\le x \le 0, \\ 2x &\le y \le 0, \\ 0 &\le z \le 4 - y^2. \end{aligned}$$

These lead to the integrals for the regions of

$$\int_0^1 \int_0^{2x} \int_0^{4-y^2} f(x,y,z)dzdydx = \int_0^1 \int_0^{4-4x^2} \int_{2x}^{\sqrt{4-z}} f(x,y,z)dydzdx$$

and

$$\int_{-1}^0 \int_{2x}^0 \int_0^{4-y^2} f(x,y,z)dzdydx = \int_{-1}^0 \int_0^{4-4x^2} \int_{\sqrt{4-z}}^{2x} f(x,y,z)dydzdx.$$

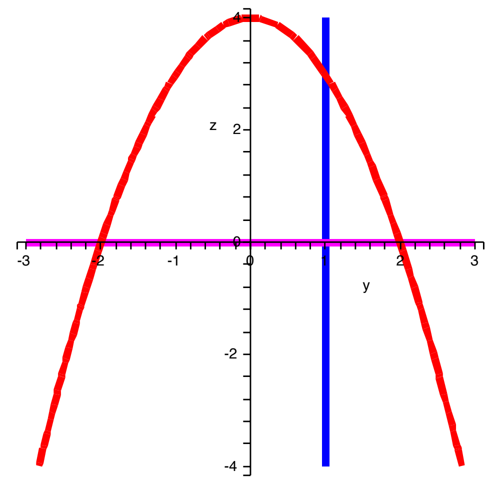

FIGURE 3. Bounding functions with $x = \frac{1}{2}$