1. (4 points) Prove the following statement by induction. For all $n \geq 1$,

$$1 + 2 + 3 + \cdots + (n-1) + n + (n-1) + \cdots + 3 + 2 + 1 = n^2$$

2. (4 points) Prove the following statement by induction. For all $n \geq 2$,

$$n! < n^n$$

3. (4 points) The sequence b_n is given by the recursive definition $b_1 = 1$ and $b_n = 4b_{n-1} + 1$ for $n \geq 2$. Carefully develop an explicit formula for b_n.

4. (4 points) Find the coefficient of $x^{28}y^{222}r^{103}s^{147}$ in $(x + y + r + s)^{500}$.

5. (4 points) The sequence a_n is given by the recurrence $a_1 = 5$ and $a_n = a_{n-1} + 3$ for $n \geq 2$. Carefully explain how to find $a_1 + \cdots + a_{500}$.

6. (4 points) Alice is paving a walkway that will be 2ft by $n$ft. She has two types of pavers available to her, an "L" shaped paver that is a total of 3 square feet as well ans a square paver that is a total of 1 square foot. Develop a *recursive* solution for the number of ways Alice can pave the walkway. (Hint: If $n = 1$, there is 1 way, if $n = 2$ there are 5 ways, and if $n = 3$ there are 11 ways.)

7. (4 points) For each of the following expressions find the coefficient of x^6y^{12}.

   (a) $\left(x^2 + y^3\right)^7$

   (b) $\left(\frac{x}{y} + y^3\right)^{10}$

   (c) $\left(x^2 + y^3\right)^{18}$

   (d) $\left(\frac{x}{y} + y^3\right)^{12}$

 Points earned:_______