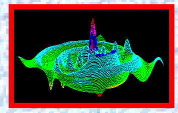
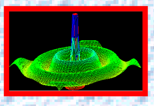
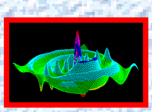


Center for Waves in Stratified Media

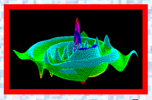
Introduction



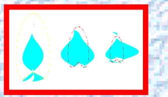
The Scattering is a natural phenomenon. Looking at the sky, we see scattered sunlight. An important fact in wave scattering is that a wave is not scattered when it traverses a homogeneous medium. Only inhomogeneities cause scattering. Therefore, scattering is also a tool to explore inhomogeneities in a medium.



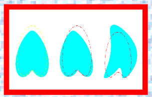
Scattering theory is concerned with the effect an inhomogeneity has on an incident wave or particle. In general, there are two kinds of problems in scattering: direct scattering and inverse scattering. We consider the total wave field as the sum of an incident wave and a scattered wave. The direct scattering problem is to determine the scattered wave from the knowledge of the incident wave and the known inhomogeneity. The inverse scattering problem, which is more interesting and more challenging, is to determine the unknown inhomogeneity from the knowledge of the incident wave and the measured scattered wave.



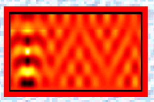
The direct scattering problem has been studied for many years. The problems are formulated as a boundary value problem, i.e., an equation called partial differential equation along with a set of boundary conditions. Solving the problem we can predict how waves propagate in a given medium. However, scientists can obtain solutions analytically only for very simple media. For inverse problems, analytic solutions are almost impossible.



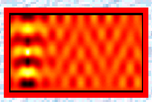
This situation was dramatically changed in the last a few decades due to the development of powerful computers. Some stable numerical methods for solving complicate boundary problems have been developed. Solving corresponding inverse problems is possible. These lead to extremely active study of the inverse scattering problems.



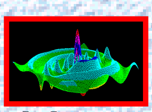
Our research is to investigate systematically the inverse scattering of acoustic and electromagnetic waves by obstacles or inhomogeneity in a stratified host medium. By stratified host medium we mean that the refractive index (a quantity for describing the inhomogeneity of the medium) of the medium is a function of one variable except in a bounded domain.



The basic problems we consider are the inverse scattering of time-harmonic acoustic or electromagnetic waves by a penetrable non-stratified inhomogeneous medium of compact support and by a bounded impenetrable obstacle in a stratified host medium. We also study inverse scattering problems in a fluid-elastic/fluid-poroelastic sediment. The problems can be generally stated as following: given the scattered waves for several incident waves with different incident directions and different modes, determine the shape of the scatterer or the inhomogeneous refractive index function.



The methodology used by Dr. R. Gilbert and Dr. Y. Xu for the reflecting sea bottom was based on finding an operator which produced the far-field from a scattered wave. This method can be generalized to more complicated problems.



The Significance of our research may be seen from different aspects. In the application aspect, our research is motivated by the need to reconstruct the shape of scattering object and determine its refractive index function from scattered data in a stratified host medium. This need can be found in many pressing issues, including the following.

1. Environmental protection.

   Temperature is a major factor to affect the refractive index of a medium. Global or large area change of temperature will affect the propagation of acoustic waves in oceans. Measuring the propagation and scattering of acoustic waves may provide an important way to determine the long term trend of temperature change. Our research can provide theoretical analysis and computer simulations for determination of the refractive index from measured acoustic waves.}

2. Exploration of oceans.

   The oceans cover more than half of the earth's surface. Their depths are filled with an unlimited variety of things we are interested in detecting, measuring, interpreting, analyzing, modeling, exploiting, and preserving. Sound wave is the only energy form that can propagate effectively in water. A better understanding of the inverse scattering of underwater sound waves will greatly improve ocean survey systems.

3. Remote sensing of objects under ice cover.

   Electromagnetic waves are used in remote sensing of areas with ice cover. Inverse scattering theory of electromagnetic waves will provide more effective algorithms in computerized detection.

Home Page **Direct Scattering**

Direct Scattering

We consider an inverse obstacle scattering problem for acoustic waves in a stratified medium. Consider a bounded obstacle described by a domain imbedded in a stratified medium.

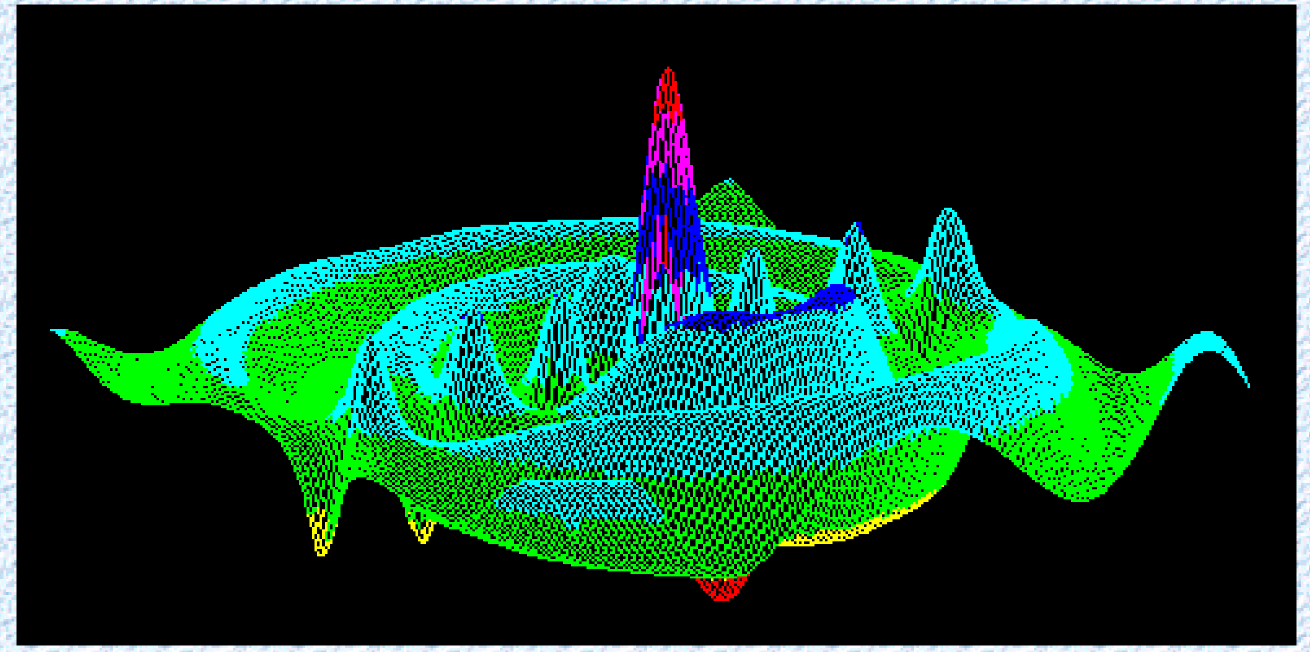
By stratified medium here we mean that the refraction index of the medium is a function of one variable.

Suppose we send in a number of incident waves, and measure the scattered waves in one or more location. Can we determined (reconstruct) the shape of the obstacle (scatterer)? Many similar inverse problems in wave propagation assume that the medium is homogeneous. However, in a stratified medium sound waves may be trapped by acoustic ducts and caused to propagate horizontally. Therefore, the scattered energy flux is not spread out spherically. Instead there are free-wave far-field and guided-wave far-field. (See Waves in a homogeneous medium for a demonstration of continuous waves from a point source in a homogeneous medium, and Waves in a stratified mediumfor a demonstration of continuous waves from a point source in a stratified medium, respectively.)

Due to the nature of stratified medium, some results which are valid for inverse scattering in a homogeneous medium may not be true for a stratified medium. For example, a scatterer is determined uniquely by far-field patterns in an open set corresponding to infinitely many linearly independent incident waves. This may not be true for stratified medium, unless the far-field is detected in a window large enough to contain both free-waves and guided-waves. Or two open sets are needed; one for free-wave far-field and one for guided-wave far-field.

Our research concerns the problems of determining the shape of scatterer from incomplete far-field data in a stratified medium. We have developed a theory for scattering of acoustical waves in a stratified medium. Theoretical analysis shows that incident waves and scattered waves have a reciprocal relation which provides a way to compensate the incomplete far-field data.

Propagating Wave in a Stratified Medium



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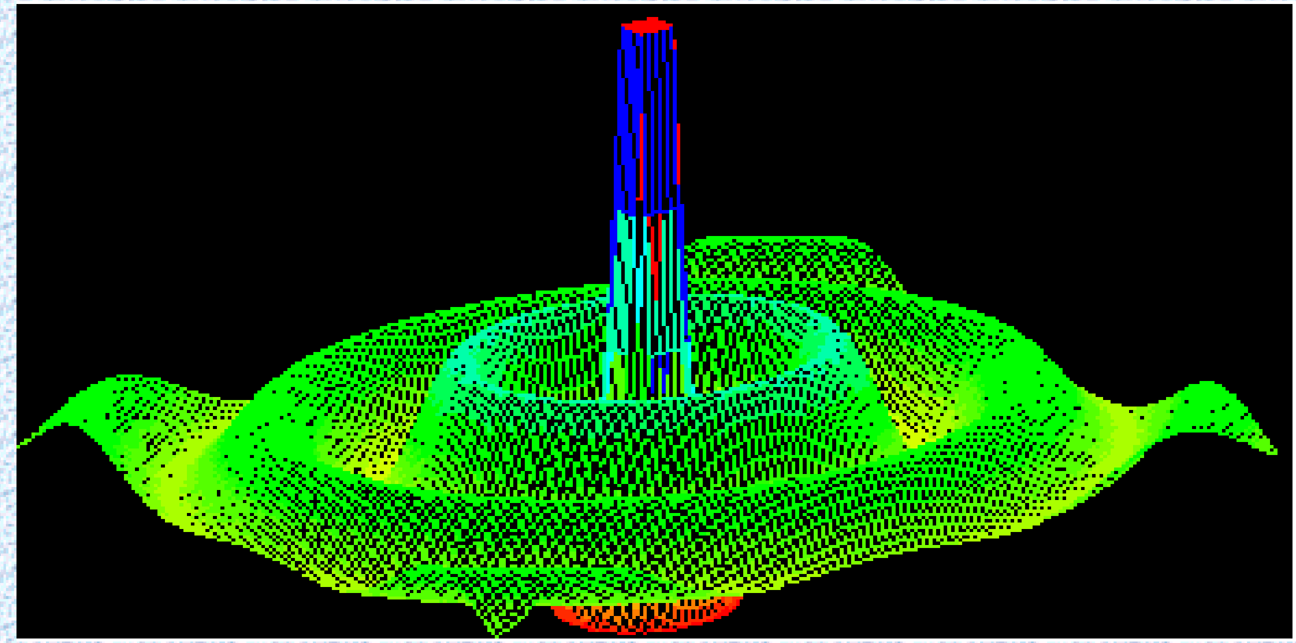
Propagating Wave in a Homogeneous Medium



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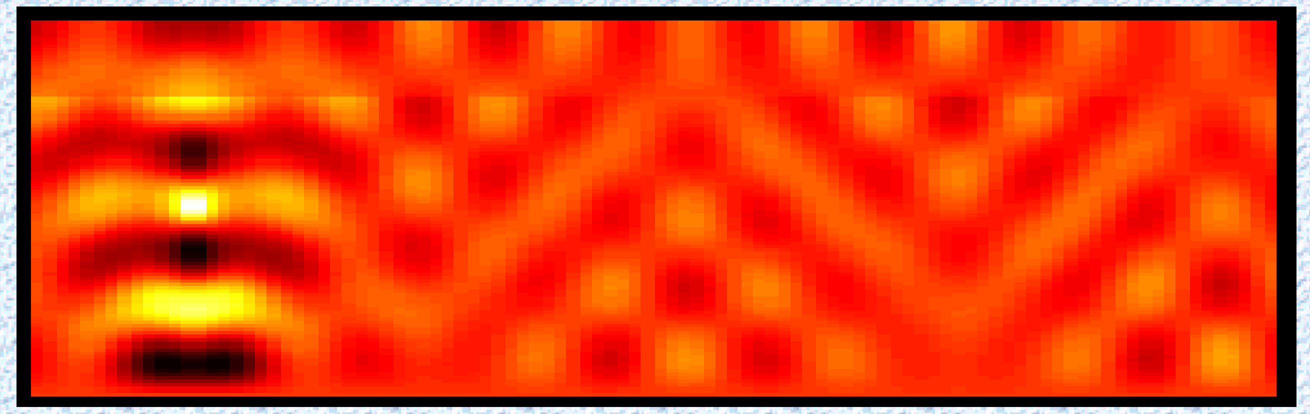
Propagation in a Two-layer Medium



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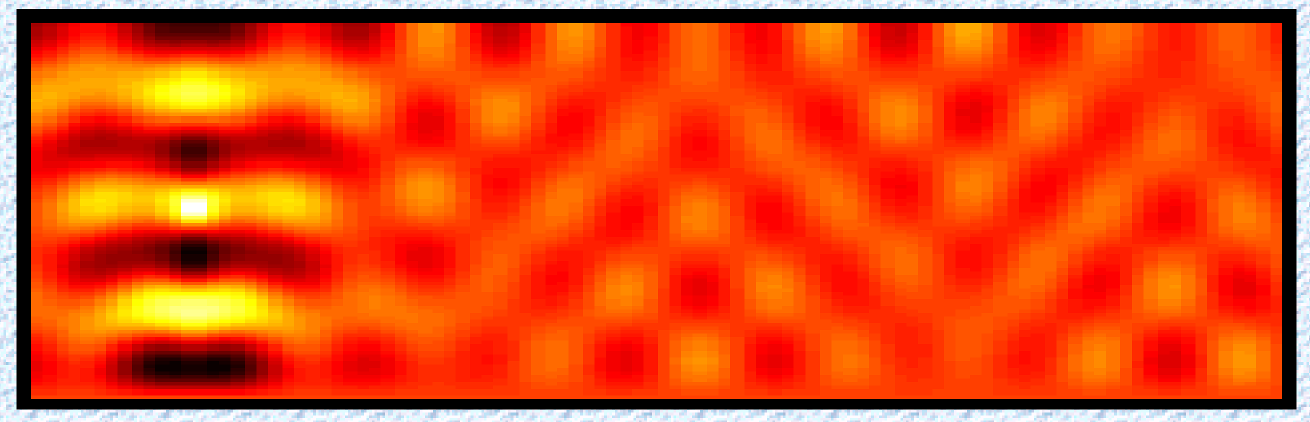
Propagation in a Homogeneous Waveguide



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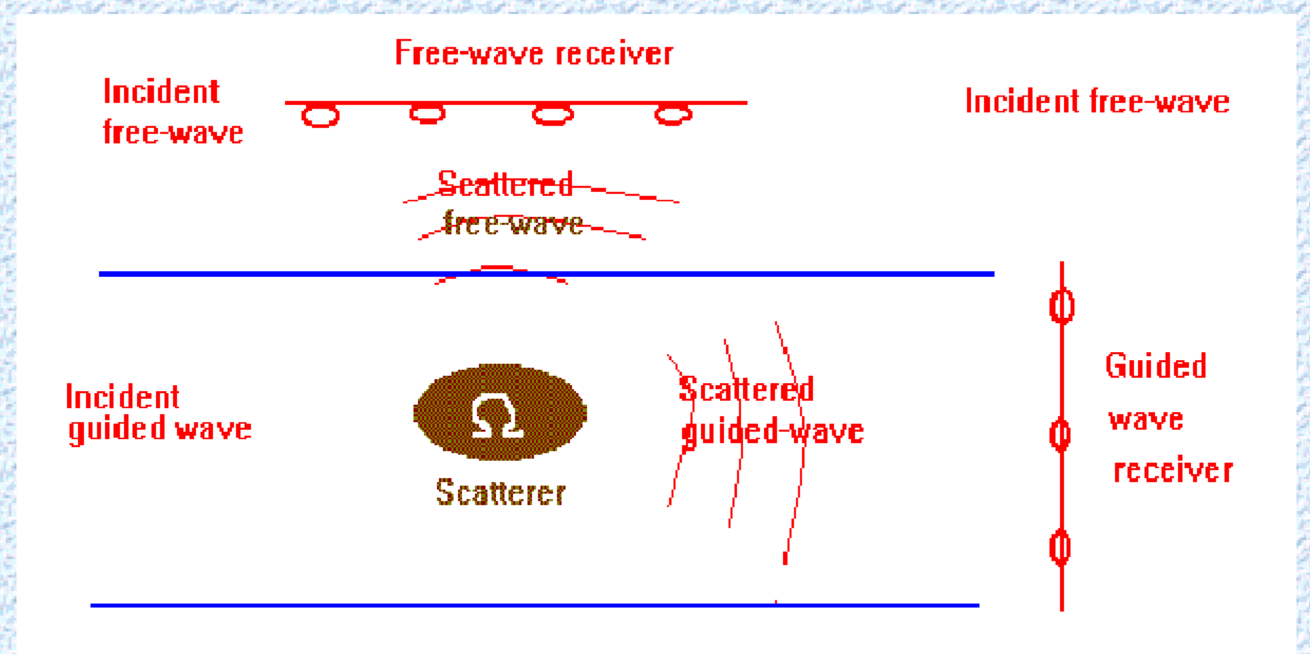
This page is best viewed with a Netscape 3.0 version browser.

Inverse Scattering

We outline the inverse scattering theory for acoustic waves in a perturbed stratified medium developed by Dr. Yongzhi Xu .

Consider wave scattering by an obstacle in a stratified medium shown in Waves scattered by an obstacle.



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The shape determination problem we consider is that given information on incident waves and far-field patterns of scattered waves, determine the shape of the scatterer.

Computer simulation has been performed to reconstruct the shape of a 2-D scatterer using incomplete far-field data for different incident waves.

The scattered far-field data is obtained by solving the direct scattering problem using an approximate boundary integral method.

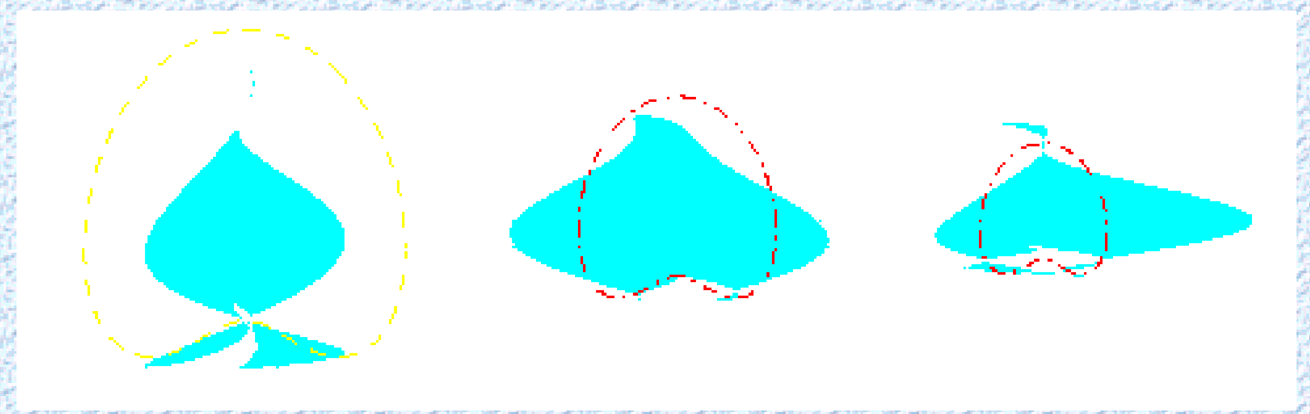
To reconstruct the shape of the scatterer, we use only the scattered free-wave far-field data corresponding to different incident waves. The inverse problem is solved by a minimization algorithm developped by Xu. Our results are shown in Figure A and Figure B . The solid curve outlines the original scatterer. The reconstruction is shown by the shaded area.

In Figure A , we plot three shape reconstruction results for incident waves chosen from only guided-mode or only downward free-waves.

In Figure B (a), one guided mode is used. In Figure B (b), two guided modes are used. In Figure B (c) one downward free-wave is used. Obviously, the results in Figure A are very poor shape reconstruction results. The results in Figure B are much better reconstructions.

These results, consistent with the theory, show that carefully chosen incident waves can greatly improve the quality of shape reconstruction in a stratified medium.

Shape Reconstruction (1)



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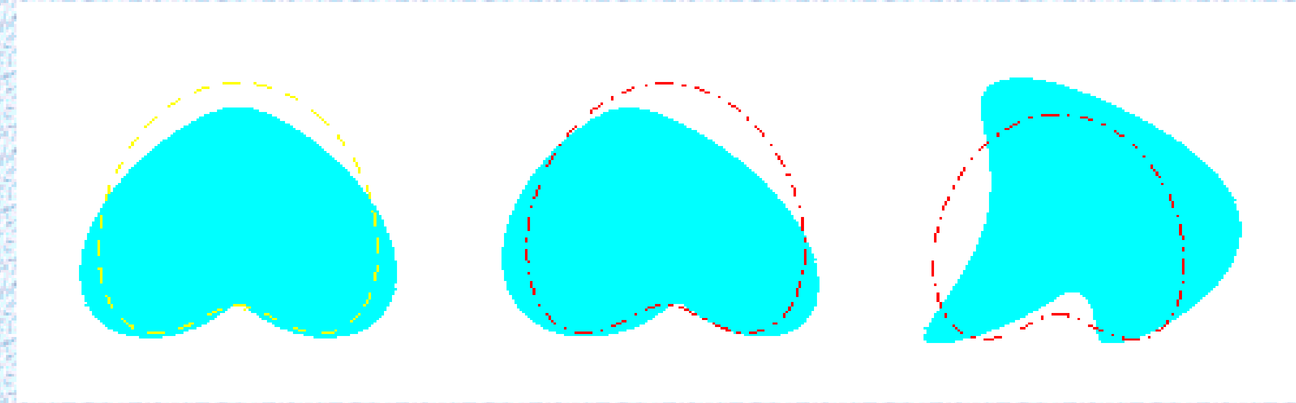
Shape Reconstruction (2)



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Direct Scattering Introduction

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