Math 205/Test 3

SHOW ALL WORK!!! NO CREDIT OTHERWISE!!!
MORE PROBLEMS ON THE BACK!!!
Each problem is worth 10 pts.

(1) Let \( f(x) = \ln(e^x + \tan(x)) \). Find \( f'(x) \).

(2) Find \( y' \) using logarithmic differentiation where
\[
y = \frac{(x^2 - 3x)^{10} (\cos(x) + 21 + x^2)^9}{(x + 3)^4 (x^3 - x + 7)^5}.
\]
(3) Two cars start moving from the same point. One travels south at 60 mph and the other travels west at 25 mph. At what rate is the distance between them increasing two hours later?

(4) Let $f(x) = x^4 - x^2 + 10$. Find the absolute max and the absolute min of $f$ over the interval $[-1, 1]$. 

(5) Let \( f(x) = 3x^5 - 5x^3 \). Find where \( f \) is increasing, decreasing, concave up, concave down. Find the local extrema as well as the inflection points of \( f \). Then, sketch the graph of \( f \).
(6) • State the mean value theorem.

• Use the mean value theorem to show that $3x + e^x - 100$ has at most one real root.

(7) Use L’Hospital’s rule to evaluate $\lim_{x \to 0} \frac{x - \sin(x)}{x^4}$. 
(8) Use L’Hospital’s rule to evaluate \( \lim_{x \to \infty} \frac{2x}{x + \ln(x)} \).

(9) If 1200 \( cm^2 \) of material is available to make a box with a square base and an open top, find the largest possible volume of the box.

(10) **Bonus.** Find the antiderivative of \( f(x) = x^2 e^x \). Show all trials, steps, etc. No credit otherwise.