Mathematics 105 Quiz #4 Spring 2010 Answer Key

\[ F = P \left(1 + \frac{r}{n}\right)^{nt} \]

\[
FA = \frac{PMT \left( (1 + i)^m - 1 \right)}{i} \quad PV = \frac{PMT \left( 1 - (1 + i)^{-m} \right)}{i}
\]

1. **Version a:** Mary's bank offers an account that pays 4.8% compounded monthly. Suppose she deposits $80 at the end of each month for the next 20 years.

   (a) What will be the total value of her account just after the last deposit? Need FA
   \[
i = \frac{0.048}{12} = 0.004 \quad m = 20 \times 12 = 240 \quad PMT = 80
   \]
   \[
   FA = 80 \left( 1.004240 - 1 \right) / 0.004 = 32,134.00
   \]
   (b) How much interest will she have earned at the end of the 20 years? The FA includes her total payments and the interest on those payments, so Interest Amount = FA - total payments = $32,134.00 - 240 x $80 = $32,134.00 – $19,200 = $12,934.00

**Version b:** Mary's bank offers an account that pays 5.4% compounded monthly. Suppose she deposits $120 at the end of each month for the next 15 years.

(a) What will be the total value of her account just after the last deposit? Need FA
\[
i = \frac{0.054}{12} = 0.0045 \quad m = 15 \times 12 = 180 \quad PMT = 120
\]
\[
FA = 120 \left( 1.0045^{180} - 1 \right) / 0.0045 = 33,168.72
\]
(b) How much interest will she have earned at the end of the 20 years? FA - total payments = $33,168.72 - 180 x $120 = $33,168.72 – $21,600 = $11,568.72

**Version c:** Mary's bank offers an account that pays 5.7% compounded monthly. Suppose she deposits $150 at the end of each month for the next 25 years.

(a) What will be the total value of her account just after the last deposit? Need FA
\[
i = \frac{0.057}{12} = 0.00475 \quad m = 25 \times 12 = 300 \quad PMT = 150
\]
\[
FA = 150 \left( 1.00475^{300} - 1 \right) / 0.00475 = 99,279.60
\]
(b) How much interest will she have earned at the end of the 20 years? FA - total payments = $99,279.60 - 300 x $150 = $99,279.60 – $45,000 = $54,279.60

2. **Version a** Frank wants to buy a new car. The dealer wants him to make a $3000 down payment and take out a loan for the rest for 4 years at 7.2% annual interest compounded monthly with monthly payments of $380.

(a) What is the amount (principal) of this loan?
\[
i = 0.072 / 12 = 0.006; \quad m = 4 \times 12 = 48 \quad PMT = 380
\]
Use formula in the right box above: PV = $380 x (1 - 1.006^-48) / 0.006 = $15,807.55
(b) What is the cash value of this car deal today? $15,807.55 + $3000 = $18,807.55

**Version b:** Frank wants to buy a new car. The dealer wants him to make a $4000 down payment and take out a loan for the rest for 4 years at 7.8% annual interest compounded monthly with monthly payments of $420.

(a) What is the amount (principal) of this loan?
\[
i = 0.078 / 12 = 0.0065; \quad m = 4 \times 12 = 48 \quad PMT = 420.
\]
Use formula in the right box above: PV = $420 x (1 - 1.0065^-48) / 0.0065 = $17,230.34
(b) What is the cash value of this car deal today? $17,230.34 + $4000 = $21,230.34

**Version c:** Frank wants to buy a new car. The dealer wants him to make a $5000 down payment and take out a loan for the rest for 3 years at 6.6% annual interest compounded monthly with monthly payments of $490.

(a) What is the amount (principal) of this loan?
\[
i = 0.066 / 12 = 0.0055; \quad m = 3 \times 12 = 36 \quad PMT = 490
\]
Use formula in the right box above: PV = $490 x (1 - 1.0055^-36) / 0.0055 = $15,963.75
(b) What is the cash value of this car deal today? $15,963.75 + $5000 = $20,963.75