Mathematics 105 Quiz #5a Answer Key

\[ F = P \left(1 + \frac{r}{n}\right)^{nt} \]

- \[ FA = \frac{PMT \left(\left(1 + \frac{r}{n}\right)^{nt} - 1\right)}{i} \]
- \[ PMT = \frac{FA \times i}{\left(\left(1 + \frac{r}{n}\right)^{nt} - 1\right)} \]
- \[ PV = \frac{PMT \left(1 - \left(1 + \frac{r}{n}\right)^{-mt}\right)}{i} \]
- \[ PMT = \frac{PV \times i}{\left(1 - \left(1 + \frac{r}{n}\right)^{-mt}\right)} \]

1. Tisha is taking out a $128,000 mortgage to buy a house. The mortgage is for 30 years at 6.3% annual interest compounded monthly.

(a) Find the amount of her monthly payment.

\[ PV = 128,000 \quad m = 12 \times 30 = 360 \quad i = \frac{0.063}{12} = .00525 \]

\[ PMT = \frac{PV \times i}{\left(1 - \left(1 + \frac{i}{12}\right)^{-360}\right)} = \frac{128,000 \times 0.00525}{\left(1 - \left(1 + \frac{0.00525}{12}\right)^{-360}\right)} = \$792.29 \]

(b) In her first monthly payment, what amount goes to pay interest and what amount goes to repay principal?

Amount Interest: \[0.00525 \times 128,000 = \$672\]

Amount Principal: \[792.29 - 672 = 120.09\]

(c) If she makes 8 years of payments, what will be the balance remaining (payoff amount) on her mortgage at that time?

Payoff Amount = PV of remaining payments.
\[ m = 360 - 8(12) = 360 - 96 = 264 \text{ payments remain.} \]

Payoff Amount: \[PV = \frac{792.29 \left(1 - \left(1 + \frac{0.00525}{12}\right)^{-264}\right)}{0.00525} = \$113,036.10\]

2. John wants to accumulate $250,000 by making identical deposits at the end of each quarter for the next 20 years into an account that pays 5% compounded quarterly. What payment amount will enable him to do this?

\[ i = \frac{0.05}{4} = 0.0125 \text{ per quarter, } \quad m = 4 \times 20 = 80 \text{ payments, } \quad 250,000 = FA \]

\[ PMT = \frac{250,000 \times 0.0125}{\left((1.0125)^{80} - 1\right)} = \$1836.63 \]
1. Tasha is taking out a $178,500 mortgage to buy a house. The mortgage is for 25 years at 6.6% interest compounded monthly.

(a) Find the amount of her monthly payment.

\[ PV = 178,500 \quad m = 12 \times 25 = 300 \quad i = 0.066/12 = 0.0055 \]

\[
PMT = \frac{PV \times i}{(1+i)^m - 1} = \frac{178500 \times 0.0055}{1 - 1.0055^{-300}} = $1216.42
\]

(b) In her first monthly payment, what amount goes to pay interest and what amount goes to repay principal?

Amount Interest = 0.0055 x 178,500 = $981.75
Amount Principal = $1216.42 - $981.75 = $234.67

(c) If she makes 11 years of payments, what will be the balance remaining (payoff amount) on her mortgage at that time?

She has 25 – 11 = 14 years, or 14 x 12 = 168 payments remaining. The payoff amount = the PV of the remaining payments.

\[
PV = \frac{1216.42(1-1.0055^{-168})}{0.0055} = $133,157.22
\]

2. John wants to accumulate $200,000 by making identical deposits at the end of each quarter for the next 20 years into an account paying 5.2% compounded quarterly. What payment amount will enable him to do this?

\[ i = 0.052 / 4 = 0.013 \text{ per quarter} , \quad m = 4 \times 20 = 80 \text{ payments} , \quad $200,000 = FA \]

\[
PMT = \frac{200,000 \times 0.013}{(1.013)^{80} - 1} = $1436.21
\]
1. Tilly is taking out a $157,500 mortgage to buy a house. The mortgage is for 15 years at 6.0% interest compounded monthly.

(a) Find the amount of her monthly payment.

\[ PV = 157,500 \]
\[ m = 12 \times 15 = 180 \]
\[ i = \frac{0.06}{12} = .005 \]
\[ PMT = \frac{PV \times i}{(1 + i)^m - 1} = \frac{157500 \times 0.005}{1 - (1 + 0.005)^{180}} = 1329.07 \]

(b) In her first monthly payment, what amount goes to pay interest and what amount goes to repay principal?

Amount Interest = 0.005 \times 157,500 = $787.50

Amount Principal = $1329.07 - $787.50 = $541.47

(c) If she makes 7 years of those monthly payments, what will be the balance remaining (payoff amount) on her mortgage at that time?

She has 15 - 7 = 8 years, or \( 8 \times 12 = 96 \) payments remaining. The payoff amount = the PV of the remaining payments.

\[ PV = \frac{1329.07(1 - 1.005^{-96})}{0.005} = 101,135.87 \]

2. John wants to accumulate $150,000 by making identical deposits at the end of each quarter for the next 16 years into an account paying 5.4% compounded quarterly. What payment amount will enable him to do this?

\[ i = \frac{0.054}{4} = 0.0135 \text{ per quarter}, \quad m = 4 \times 16 = 64 \text{ payments}, \quad F_A = 150,000 \]
\[ PMT = \frac{F_A \times i}{(1 + i)^m - 1} = \frac{150,000 \times 0.0135}{(1.0135)^{64} - 1} = 1490.11 \]