Math 105 Spring 2010 Quiz 3 Answer Key

\[ F = P \left( 1 + \frac{r}{n} \right)^{nt} = P(1+i)^m \quad F = P(1+Y)^t \quad Y = \left( 1 + \frac{r}{n} \right)^n - 1 \]

Version a:

1. Assume the annual rate of inflation is 4.2% for the next 5 years. What salary five years from now will have the same purchasing power as a current salary of $48,000? Round your answer in accordance with the Rule for Accuracy of Combined Numbers

Equivalent Purchasing Power in Future = $48,000 \left( 1 + 0.042 \right)^5 = 58963.04

rounds to $59,000 since the data (rate, dollar amount) only has 2 significant digits.

2. John invests $1,000 in an account paying 4.8% annual interest compounded monthly.
   (a) How long will it be until the value of the account reaches at least $3,800? State your answer in years and months.

   \[ r = 0.048, \quad n = 12, \quad \text{so} \quad i = \frac{0.048}{12} = 0.004. \quad \text{We need to find the number} \quad m \quad \text{of monthly compoundings.} \]

   Solve \[ 3800 = 1000(1+0.004)^m \]

   Divide by $1000: \[ 3.8 = (1.004)^m \]

   Take logs: \[ \log(3.8) = \log((1.004)^m) \]

   Use Log property: \[ \log(3.8) = m \log(1.004) \]

   Divide by \log(1.004): \[ m = \log(3.8) / \log(1.004) \]

   Evaluate: \[ m = 334.4\ldots \quad \text{so it will take 335 months. This is 27 years, 1 months.} \]

   (b) How much will his investment be worth at the end of the length of time found in (a)?

   Using the \( m = 335 \) from (a), \[ F = 1000(1.004)^{335} = 3808.85 \]

3. What 2003 salary had the same purchasing power as a 1990 salary of $35,250? (In 1990 the CPI was 130.7, and in 2003 the CPI was 184.0.) Round your answer in accordance with the Rule for Accuracy of Combined Numbers.

   Salary in 2003 dollars = \left( \frac{184.0}{130.7} \right)^{35,250} = 49,625.09. \quad \text{Since the CPIs and salary have 4 significant digits, we round to $49,630.}
1. John invests $2,000 in an account paying 5.4% annual interest compounded monthly. 
   (a) How long will it be until the value of the account reaches at least $7,400? State your answer in years and months.
   
   \[ r = 0.054, \ n = 12, \ \text{so} \ i = 0.054/12 = 0.0045. \] We need to find the number \( m \) of monthly compoundings.

   Solve \[ 7400 = 2000(1+0.0045)^m \]
   Divide by $2000: \[ 3.7 = (1.0045)^m \]
   Take logs: \[ \log(3.7) = \log((1.0045)^m) \]
   Use Log property: \[ \log(3.7) = m \log(1.0045) \]
   Divide by \( \log(1.0045) \): \[ m = \log(3.7) / \log(1.0045) \]
   Evaluate: \[ m = 291.3943... \] so it will take 292 months. This is 24 years, 4 months.

   (b) How much will his investment be worth at the end of the length of time found in (a)?

   Using the \( m = 213 \) from (a), \[ F = 2000(1.0045)^{292} = 7420.15 \]

2. What 2003 salary had the same purchasing power as a 1980 salary of $25,400? (In 1980 the CPI was 82.4, and in 2003 the CPI was 184.0.) Round your answer in accordance with the Rule for Accuracy of Combined Numbers.

   \[ \text{Salary in 2003 dollars} = \left( \frac{184.0}{82.4} \right) 25,400 = 56,718.45. \] Since one of the CPIs has only 3 significant digits, we round to $56,700.

3. Assume the annual rate of inflation is 5.2% for the next 6 years. What salary six years from now will have the same purchasing power as a current salary of $58,000? Round in accordance with the Rule for Accuracy of Combined Numbers.

   \[ \text{Equivalent Purchasing Power in Future} = 58,000 \ (1 + 0.052)^6 = 78,618.08 \]
   rounds to $79,000 since the data (rate, dollar amount) only has 2 significant digits.
Version c:

1. What 2003 salary had the same purchasing power as a 1985 salary of $30,500? (In 1985 the CPI was 107.6, and in 2003 the CPI was 184.0.) Round your answer in accordance with the Rule for Accuracy of Combined Numbers.

   Salary in 2003 dollars = \( \left( \frac{184.0}{99.6} \right)^{30500} = 56,345.38 \). Since one of the CPIs has only 3 significant digits, we round to $56,300.

2. Assume the annual rate of inflation is 4.5% for the next 7 years. Seven years from now what will be the purchasing power, in today’s dollars, of a current salary of $42,000? Round your answer in accordance with the Rule for Accuracy of Combined Numbers.

   Equivalent Purchasing Power in Future = $42,000 (1 + 0.045)^7 = 57156.20

   rounds to $57,000 since the data (rate, dollar amount) only has 2 significant digits.

3. John invests $2500 in an account paying 4.2% annual interest compounded monthly. (a) How long will it be until the value of the account reaches at least $9500? State your answer in years and months.

   \( r = 0.042, n = 12, \) so \( i = 0.042/12 = 0.0035. \) We need to find the number \( m \) of monthly compoundings.

   Solve \( $9500 = $2500(1+0.0035)^m \)

   Divide by $2500: \( 3.8 = (1.0035)^m \)

   Take logs: \( \log(3.8) = \log((1.0035)^m) \)

   Use Log property: \( \log(3.8) = m \log(1.0035) \)

   Divide by log(1.0045): \( m = \log(3.8) / \log(1.0035) \)

   Evaluate: \( m = 382.095988... \) so it will take 383 months. This is 31 years, 11 months

   (b) How much will his investment actually be worth at the end of the full number of months found in (a)?

   Using the \( m = 383 \) from (a), \( F = $2500(1.0035)^{383} = $9530.05 \)